Year: Month: Date: ( ) Subject:

تمارین فصل اکستری

(1)

$f(x) = \cos^3(2x) + ax^2 + b$

$\lim\limits_{x \to 0^+} \dfrac{f(x)}{x} = 0$

$\lim\limits_{x \to 0^+} \dfrac{\cos^3(2x) + ax^2 + b}{x} = 0 \Rightarrow \dfrac{1 - 3\dfrac{(2x)^2}{2} + ax^2 + b}{x} = \dfrac{1 - 6x^2 + ax^2 + b}{x}$

$\Rightarrow b + 1 = 0$

$\Rightarrow b = -1$

$\lim\limits_{x \to 0^+} \dfrac{(a-6)x^2}{x} = (a-6)x = 0$

$\rightarrow \begin{cases} f'(x) = 2(a-6)x \\ f''(x) = 2(a-6) \end{cases} \quad \lim\limits_{x \to 0^-} \dfrac{f'(x)}{x} \overset{H.P}{=} \dfrac{f''(x)}{1} = \dfrac{2(a-6)}{1} = 2$

$\Rightarrow a - 6 = 1$

$\Rightarrow a = 7$

$a + b = 7 - 1 = 6$

(2)

$A = (-0.5, -12), B = (2.5, 6) \Rightarrow m_{AB} = 6, y = 6x - 9$

$f'(x) = \dfrac{-2a}{(2x-1)^2} = 6, \quad f(x) = \dfrac{a}{2x-1} = 6x - 9$

$\begin{cases} a = -3(2x-1)^2 \\ a = (2x-1)(6x-9) \end{cases} \Rightarrow -3(2x-1) = 6x - 9 \Rightarrow +6x - 9 = -6x + 3$

$\Rightarrow 12x = 12$

$\Rightarrow x = 1$

$a = -3 \Rightarrow f(5) = \dfrac{-3}{9} = \dfrac{-1}{3}$

(3)

$f'(1) = 2$

$f(x) = \dfrac{1 - a^2}{(ax+1)^2} \Rightarrow f'(1) = \dfrac{1 - a^2}{(a+1)^2} = \dfrac{1 - a}{a+1} = 2$; $(a+1)(a-1)$

$\Rightarrow 2a + 2 = 1 - a \Rightarrow 3a = -1 \Rightarrow a = -\dfrac{1}{3}$

$f(1) = \dfrac{1 - \frac{1}{3}}{-\frac{1}{3} + 1} = 1 \xrightarrow[y = 2x + b]{} 2(1) + b = 1 \Rightarrow b = -1$

$a - b = -\dfrac{1}{3} + 1 = \dfrac{2}{3}$

(۵)

$f(x) , g(x) \Rightarrow \sin x + \frac{1}{2}\cos x = \frac{3}{2}\sin x \Rightarrow \frac{1}{2}\cos x = \frac{1}{2}\sin x$

$\Rightarrow x = \frac{\pi}{4}$

$f(\frac{\pi}{4}) = \sin\frac{\pi}{4} + \frac{1}{2}\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4} = \frac{3\sqrt{2}}{4}$ در بازه $[0, \pi]$

$(\frac{\pi}{4}, \frac{3\sqrt{2}}{4})$ نقطه تلاقی

مشتق $f(x)$ $\Rightarrow f'(x) = \cos x - \frac{1}{2}\sin x \xrightarrow{x=\frac{\pi}{4}} \cos\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4}$ شیب خط مماس

$(\frac{\pi}{4}, \frac{3\sqrt{2}}{4}) \Rightarrow y - y_0 = m(x - x_0)$

$y - \frac{3\sqrt{2}}{4} = \frac{\sqrt{2}}{4}(x - \frac{\pi}{4})$ تلاقی با محور x $\Rightarrow y=0$ $\quad -\frac{3\sqrt{2}}{4} = \frac{\sqrt{2}}{4}(x - \frac{\pi}{4})$

$\Rightarrow -\frac{3\sqrt{2}}{4} + \frac{\pi\sqrt{2}}{16} = \frac{\sqrt{2}}{4}x \xrightarrow{\times\frac{16}{\sqrt{2}}} -12 + \pi = 4x$

$\Rightarrow x = \frac{\pi}{4} - 3$

(۶)

$f'(x) = 6x^2 - 6x - 12 = 0 \Rightarrow \begin{cases} A(-1, 8) \\ B(2, -19) \end{cases} \Rightarrow m_{AB} = \frac{-19-8}{2-(-1)} = -9$

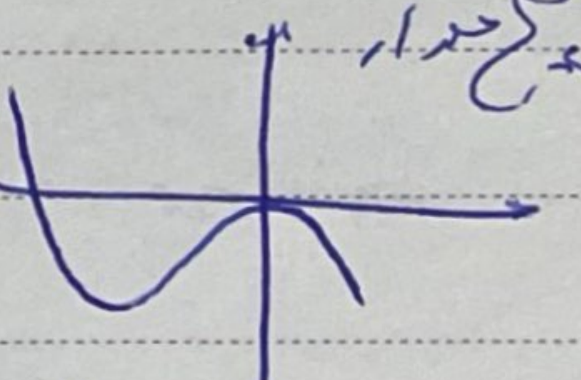
$6(x+1)(x-2)$

$f'(x) = -9 \Rightarrow 6x^2 - 6x - 12 = -9 \Rightarrow 6x^2 - 6x - 3 = 0$

$\Rightarrow \Delta > 0 \Rightarrow$ ۲ جواب

توقف

(۷)

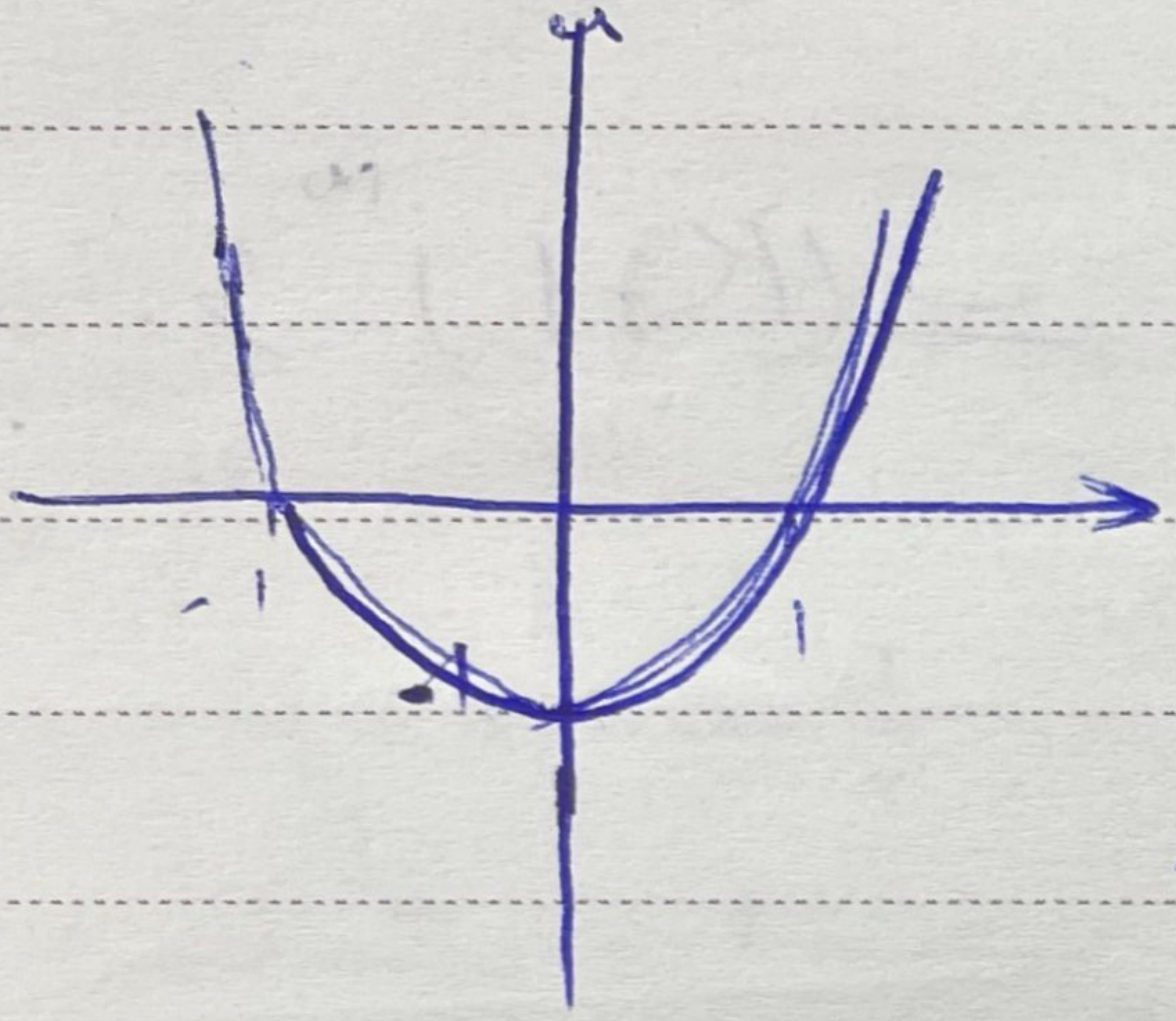


⊙ تعداد

ریشه ۲ $\Rightarrow x<0 , y>0$

x عطف $= -\frac{(k+1)}{3k} < 0 \Rightarrow R - [-1, 0]$ $\quad k<0 \Rightarrow \emptyset$

y عطف $= \frac{-(k+1)^3}{27k^2} + \frac{(k+1)^3}{9k^2} > 0 \Rightarrow (k+1)^3 > 0 \Rightarrow k > -1$



۲

$-\alpha , \alpha$ نقاط $\Rightarrow f'(-\alpha) \times f'(\alpha) = -۱$

$f'(x) = ۲x$

$\Rightarrow (-۲\alpha)(+۲\alpha) = -۱$

$\Rightarrow \alpha^۲ = \frac{۱}{۴}$

$f(\alpha) = f(-\alpha)$

$\Rightarrow \alpha^۲ - ۱ = \frac{۱}{۴} - ۱ = \frac{۱}{۴} - \frac{۴}{۴} = -\frac{۳}{۴}$

$-\frac{۳}{۴} + \frac{-۳}{۴} = -\frac{۶}{۴} = -\frac{۳}{۲}$

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(8)

$y = x^3 + ax^2 + bx - 1 \Rightarrow -4 = -1 + a - b - 1$

$\boxed{-2 = a - b}$

$y' = 3x^2 + 2ax + b \Rightarrow -4 = 3 - 2a + b \Rightarrow \boxed{-2a + b = -7}$

$f(-1) = -4 \quad , \quad f'(-1) = -4$

$\begin{cases} -2a + b = -7 \\ a - b = -2 \end{cases}$

$-a = -9 \Rightarrow a = 9$, $b = 11$ $\Big\}$ $\boxed{\dfrac{a}{b} = \dfrac{9}{11}}$

(9)

$f'(0) = 0$

$f(0) = 4$

$f(x) = x^3 + ax^2 + bx + c \xrightarrow{f(0)=4} c = 4$

$f'(x) = 3x^2 + 2ax + b \xrightarrow{f'(0)=0} b = 0$

$f'(x) = 3x^2 + 2ax = 0 \Rightarrow x(3x + 2a) = 0 \quad x = -\dfrac{2a}{3}$

$f\left(-\dfrac{2a}{3}\right) = 0 \Rightarrow \left(-\dfrac{2a}{3}\right)^3 + a\left(-\dfrac{2a}{3}\right)^2 + 4 = 0$

$\dfrac{4}{27}a^3 = -4 \Rightarrow a^3 = -27 \Rightarrow a = -3 \Rightarrow \text{min} = -\dfrac{2a}{3} = 2$

(10)

$f(x) = x^4 - 6x^2 + 5$

$f'(x) = 4x^3 - 12x$

$f''(x) = 12x^2 - 12$

y	-4	0	5	0	-4
x	$-\sqrt{3}$	-1	0	1	$+\sqrt{3}$
f(x)	↘	↗	↗	↘	↗
f'(x)	- 0 +		+ 0 -		- 0 +
f''(x)	+	0	-	0	+

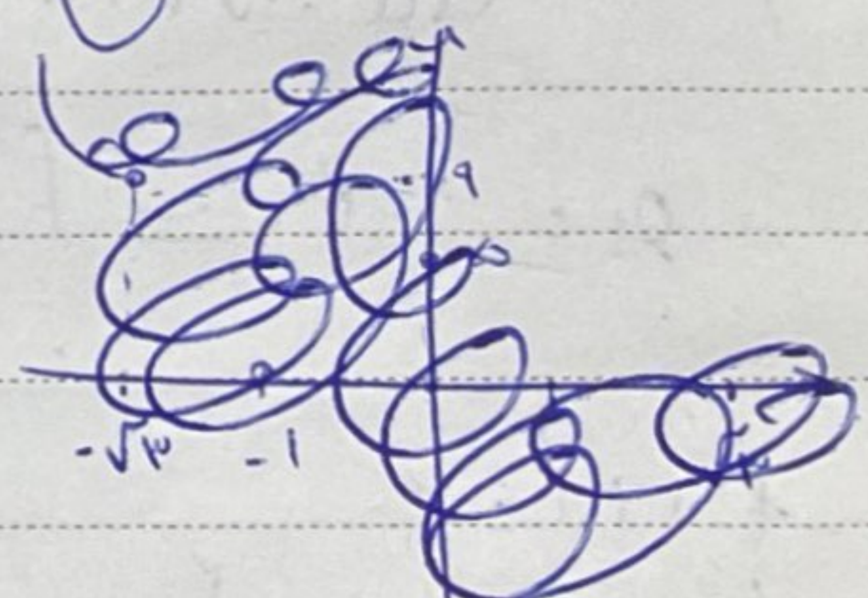
زاویه بین

$x(4x^2 - 12) = 0$

$x = 0$

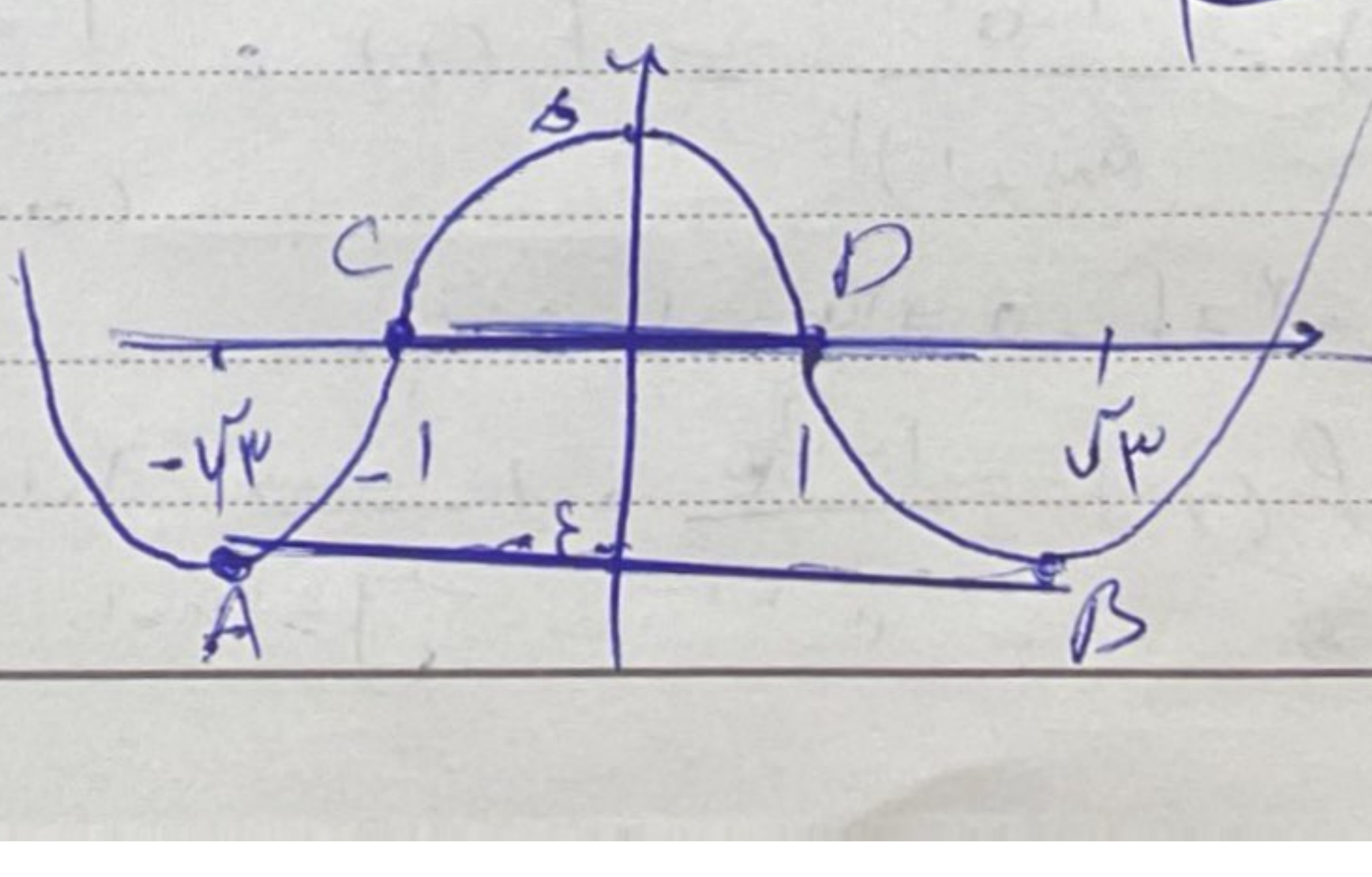
$4x^2 - 12 = 0 = 4x^2 = 12 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$

$12x^2 - 12 = 0 = 12x^2 = 12 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$



$C = 1 \quad D = -1$

$B = -\sqrt{3} \quad A = +\sqrt{3}$



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