۱۷/۵

مسابقات الگسنی — دوازدهم ریاضی A

$\lim_{x\to 0^-} \frac{f(x)}{x} = 2 \to \lim_{x\to 0^-} \frac{-6\sin(2x)\cos^2(2x)+2ax}{x} = 2 \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0^-} \frac{-6\times 2x+2ax}{x} = 2 \to \lim_{x\to 0^-} \frac{(2a-12)x}{x} = 2 \to 2a-12=2 \to \boxed{a=7}$

۱- $f(x) = \cos^3(2x) + ax^2 + b$ $\quad a+b=?$

$f'(x) = 3\cos^2(2x)(2)(-\sin(2x)) + 2ax = -6\cos^2(2x)(\sin(2x)) + 2ax$

$\lim_{x\to 0^+} \frac{f(x)}{x} = \left(\frac{0}{0}\right)$ $\quad \lim_{x\to 0^-} \frac{f'(x)}{x} = (2)$

$a+b=6$

$f(0)=0 \Rightarrow \cos^3(0) + a(0) + b = 0 \Rightarrow \boxed{b=-1}$ ✓

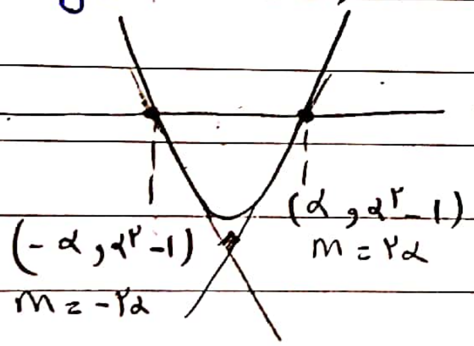
$\lim_{x\to 0^-} \frac{-6\cos^2(2x)\sin(2x)+2ax}{x} = 2 \Rightarrow a+b$

* هم‌ارزی: $\cos x = 1 - \frac{x^2}{2} \Rightarrow \cos 2x = 1 - \frac{4x^2}{2} = 1 - 2x^2$

$\lim_{x\to 0^-} \frac{-6(1-2x^2)^2(2x) + 2ax}{x} = 2$

$\lim_{x\to 0^-} \frac{2x\left(-6(1-2x^2)^2 + a\right)}{x} = 2 \Rightarrow 2a = 2 \Rightarrow \boxed{a=1}$

۲- $y = x^2 - 1 \to f'(x) = 2x$



$(2\alpha)(-2\alpha) = -1$

$4\alpha^2 = 1 \to \alpha^2 = \frac{1}{4} \to \alpha = \pm\frac{1}{2}$

$\alpha^2 - 1 = \frac{1}{4} - 1 = -\frac{3}{4}$

$\boxed{-\frac{3}{4}: \text{عرض نقطه}}$ ✓

۳- $f(x) = \frac{x}{2x-1} \to f'(x) = \frac{-2a}{(2x-1)^2}$ $\quad f(a)=?$

$\left.\begin{matrix}(-0/5, -12) \\ (2/5, 6)\end{matrix}\right\} m = \frac{18}{3} = 6 \Rightarrow y = 6x - 9$

$f(x) = 6x - 9, \quad f'(x) = 6$

$\boxed{f(a) = \frac{-3}{9}}$ ✓ جواب نهایی

$\frac{a}{2a-1} = 6a - 9$

$24a^2 - 24a + 6 = -2a$

$\to 12a^2 - 24a + 9 = a \leftarrow \boxed{4a^2 - 1 = -a} \Rightarrow 16a^2 - 24a + 8 = 0$

$a = \frac{1}{2} \to a = 0$ ✗

$a = 1 \to a = -3$ ✓

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۴-

$f(x)$: $y = 2x + b$ $g(x)$: $y = \frac{x + a}{ax + 1}$ $a - b = ?$

$f(1) = g(1)$ $f'(1) = g'(1)$

$2 + b = \frac{1 + a}{a + 1} = 1$ $2 = \frac{1 - a^2}{(ax + 1)^2} = \frac{1 - a^2}{(a+1)^2}$ (۲)

$b = -1$ $2(1 + a)^2 = 1 - a^2$

$2(1 + a^2 + 2a) = 1 - a^2$

$\Rightarrow 2 + 2a^2 + 4a = 1 - a^2 \Rightarrow 3a^2 + 4a + 1 = 0$

$a = -1$ ✗ $a = -\frac{1}{3}$ ✓ $\Rightarrow a - b = \frac{-1}{3} - (-1) = \frac{2}{3}$ ✓

۵-

$y - \frac{3\sqrt{2}}{4} = \frac{\sqrt{2}}{4}(x - \frac{\pi}{4}) \xrightarrow{y=0} -\frac{3\sqrt{2}}{4} = \frac{\sqrt{2}}{4}(x - \frac{\pi}{4}) \rightarrow x = \frac{\pi}{4} - 3$

(۱/۵)

$f(x) = \sin x + \frac{1}{2}\cos x$ $g(x) = \frac{3}{2}\sin x$

$* f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4} = \frac{3\sqrt{2}}{4}$

$\sin x + \frac{1}{2}\cos x = \frac{3}{2}\sin x$

$\frac{1}{2}\cos x = \frac{1}{2}\sin x \Rightarrow \cos x = \sin x \xrightarrow{x \in [0, \pi]} x = \frac{\pi}{4}$

$f'(x) = \cos x - \frac{1}{2}\sin x$ $(\frac{\pi}{4}, \frac{3\sqrt{2}}{4})$

$f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4}$ $y - \frac{3\sqrt{2}}{4} = \frac{\sqrt{2}}{4}(x - \frac{\pi}{4})$

$y = \frac{\sqrt{2}}{4}x - \frac{\sqrt{2}\pi}{16} + \frac{3\sqrt{2}}{4}$

محل برخورد با محور x : $\frac{\sqrt{2}}{4}x = \frac{\sqrt{2}}{4}\pi - \frac{3\sqrt{2}}{4} \longrightarrow x = \pi - 3$

۶-

$f(x) = 2x^3 - 3x^2 - 12x + 1$ f نسبی ext B, A

$f'(x) = 6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$ $\begin{cases} x = -1 & f \\ x = 2 & f \end{cases}$

x		-1		2	
f'	+	0	−	0	+
f	↗	8 (max نسبی)	↘	−19 (min نسبی)	↗

(۲)

$A(-1, 8)$ $m_{AB} = \frac{27}{-3} = -9 \longrightarrow f'(x) = -9$

$B(2, -19)$

$6x^2 - 6x - 12 = -9 \rightarrow 6x^2 - 6x - 3 = 0 \rightarrow 2x^2 - 2x - 1 = 0$

$x = \frac{1 \pm \sqrt{3}}{2} \longrightarrow$ ✓ (۲ نقطه) ✓

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۷ـ

$y = kx^3 + (k+1)x^2 = x^2(kx + k + 1) \quad (k < 0,\ k \in \mathbb{Z})$

$x_I = \dfrac{-(k+1)}{3k}$ $\quad y_I > 0$ $\quad$ $x < 0 \Rightarrow y > 0$

$kx + k + 1 = 0$

$kx = -k - 1$

$x = \dfrac{-k-1}{k} > 0$

$x_I < 0 \rightarrow \dfrac{-(k+1)}{3k} > 0$

$k+1 \rightarrow -1$, $\quad \dfrac{k+1}{3k} < 0$, $\quad 3k \rightarrow 0$

$k < -1,\ k > 0 \quad (I)$

$k > -1 \quad (II)$

$(I) \cap (II) \rightarrow k > 0 \rightarrow$ هیچ مقدار صحیح و منفی نیست $[-1, 0)$

$\Rightarrow k = -1 \rightarrow$ یک مقدار

$[0, +\infty) \cup (-\infty, -1]$

۸ـ

$y = x^3 + ax^2 + bx - 1 \qquad (-1, -4) \qquad \dfrac{a}{b} = ?$

$y' = 3x^2 + 2ax + b \qquad f(-1) = -4 \rightarrow -1 + a - b - 1 = -4$

$f'(-1) = 0 \Rightarrow 3 - 2a + b = 0 \qquad \boxed{a - b = -2}$

$-2a + b = -3 \Rightarrow \qquad \boxed{-2a + b = -3}$

$\dfrac{a}{b} = \dfrac{3}{5}$

$\dfrac{a}{b} = \dfrac{5}{7} \checkmark \Leftarrow \quad -a = -5 \Rightarrow a = 5,\ b = 7$

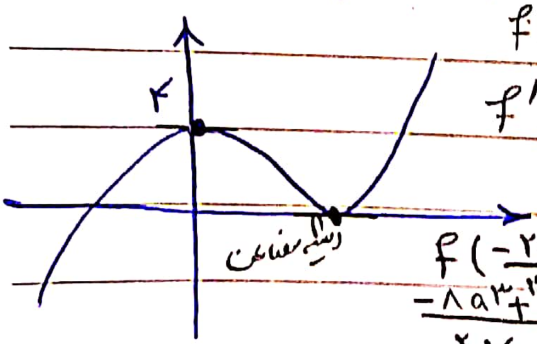
عطف $x = \dfrac{-b}{3a} \rightarrow x = \dfrac{-a}{3} \rightarrow$

$\dfrac{-a}{3} = -1 \rightarrow \boxed{a = 3}$

$-4 = -1 + 3 - b - 1 \rightarrow \boxed{b = 5}$

۹ـ

$f(x) = x^3 + ax^2 + bx + c \rightarrow f'(x) = 3x^2 + 2ax + b$



$f(0) = 4 \rightarrow c = 4$

$f'(0) = 0 \rightarrow b = 0$

نسبی Min $= \dfrac{-2a}{3} = \dfrac{-2(-3)}{3} = 2$:طول نهایی

$f'(x) = 3x^2 + 2ax = 0 \quad x(3x + 2a) = 0 \quad x = 0,\ x = \dfrac{-2a}{3}$

$f\left(\dfrac{-2a}{3}\right) = 0 \rightarrow \dfrac{-8a^3}{27} + \dfrac{4a^3}{9} + 4 = 0$

$\dfrac{-8a^3 + 12a^3}{27} = -4 \Rightarrow 4a^3 = -4 \times 27 \Rightarrow a = -3$

۱۰ـ

$f(x) = x^4 - 6x^2 + 5 \qquad f(\sqrt{3}) = 9 - 18 + 5 = -4$

$f(-\sqrt{3}) = -4$ $\quad$ نسبی Min $\leftarrow B, A$

$f'(x) = 4x^3 - 12x = 0 \qquad A(\sqrt{3}, -4),\ B(-\sqrt{3}, -4) \quad y = -4 = m$ $\quad$ عطف $\leftarrow D, C$

$4x(x^2 - 3) = 0$

$x = 0 \quad x = \sqrt{3} \quad x = -\sqrt{3}$

* دو خط با هم موازی اند

$-\sqrt{3} \quad 0 \quad \sqrt{3}$

$- \quad + \quad - \quad +$

Min $\quad$ max $\quad$ Min

$f''(x) = 12x^2 - 12 = 0$ $\quad$ پس زاویه بین آنها صفر است

$x = \pm 1$

$f(1) = 0,\ f(-1) = 0 \quad y = 0 = m'$

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