(سارا امینی راد) ۱۶٫۲۵

۱)

$\cos^3(2x)+ax^2+b \to f'(x) = 3\times 2(-\sin 2x)\times \cos^2 2x + 2ax$

$-6\sin 2x\cos^2 2x + 2ax \xrightarrow{hop} \dfrac{-6(2\cos 2x\times \cos^2 2x + \sin 2x \times 2\cos 2x \times (-2\sin 2x)) + 2a}{} = 2$

$\to 2a = 2 \Rightarrow a = 1 \qquad \lim_{x\to 0^+}\dfrac{f(x)}{x} = 1 \to \dfrac{\cos^3(2x)+ax^2+b}{x} \to b = -1 \qquad \boxed{a+b=0}$

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۲)

$y = x^2-1 \to y' = 2x \qquad 2x(-2x) = -1 \to x = \pm\frac{1}{2} \to y = \frac{1}{4}-1 = \frac{3}{4}$

$-\frac{3}{4}\times 2 = -\frac{3}{2}$

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$\lim_{x\to 0^-}\dfrac{f'(x)}{x} = 2 \to \lim_{x\to 0^-}\dfrac{-6\sin(2x)\cos^2(2x)+2ax}{x} = 2 \xrightarrow{\text{هم‌ارزی } \sin} \lim_{x\to 0^-}\dfrac{-6\times 2x+2ax}{x} = 2$

$\to \lim_{x\to 0^-}\dfrac{(2a-12)x}{x} = 2 \to 2a-12 = 2 \to \boxed{a=7} \qquad a+b=6$

سوال ۱

۳)

$\dfrac{6+12}{\frac{2.5+0.5}{3}} = 6 \to 6x-9 = y \qquad f(x) = \dfrac{a}{2x-1} \to f'(x) = \dfrac{-a\times 2}{(2x-1)^2} = 6$

$6x-9 = \dfrac{a}{2x-1}$

$a = -3(2x-1)^2 = (6x-9)(2x-1)$, $2x-3$

$-4x^2+4x-1 = 4x^2-8x+3 \to 8x^2-12x+4=0 \to x = 1, \cancel{\tfrac{1}{2}} \to a = -3$

$f(5) \to \dfrac{-3}{10-1} = -\dfrac{1}{3}$

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۴) ۱٫۷۵

$y = \dfrac{x+a}{ax+1}$ ($1+a$, $1+a$) $\to \dfrac{(1+a)-a(1+a)}{(1+a)^2} = 2 \to a = \cancel{1}, -\frac{1}{3}$

$y = \dfrac{x-\frac{1}{3}}{-\frac{1}{3}x+1} = 1 \to y = 2x+b \to 1 = 2+b \to b = -1$

$a-b = \frac{2}{3}$

۵)

$\frac{3}{2}\sin x = \sin x + \frac{1}{2}\cos x \to \sin x = \cos x \to x\in[0,\pi] \quad x = \frac{\pi}{4} \to f(x) = \frac{3\sqrt{2}}{4}$

$f'(x) = \cos x - \frac{1}{2}\sin x \xrightarrow[x=\frac{\pi}{4}]{} \frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4} \qquad y = \frac{\sqrt{2}}{4}x+b$

$\frac{3\sqrt{2}}{4} \quad \frac{\pi}{4} \qquad -\frac{\pi\sqrt{2}}{16}+\frac{3\sqrt{2}}{4}$

$\frac{\sqrt{2}}{4}x = \frac{\pi\sqrt{2}}{16} - \frac{3\sqrt{2}}{4} \to x = \frac{\pi}{4}-3$

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۶)

$f'(x) = 6x^2-6x-12 \to 6(x-2)(x+1) \to A=(2,-19)\ B(-1,8) \to m_{AB} = -\frac{27}{3} = -9$

$\to 6x^2-6x-12 = -9 \to 2x^2-2x-1 \xrightarrow[\Delta>0]{}$ ۲ نقطه وجود دارد

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۷) ۰٫۵

$y' = 3Kx^2+2(K+1)x \to y'' = 6Kx+2(K+1) \to 0 = 6Kx+2K+2$

$y = Kx^3+(K+1)x^2 \to y' = 3Kx^2+2(K+1)x \to y'' = 6Kx+2(K+1) = 0$

$x = \dfrac{-K-1}{3K} \xrightarrow[\text{ناحیه دوم}]{\text{نقطه عطف در}} -\dfrac{K+1}{3K} < 0 \to \begin{array}{c|c|c} & -1 & 0 \\ - & + & - \end{array} \to \boxed{K<-1,\ K>0\ (I)} \qquad (I)\cap(II) \to K>0$

$\to -\dfrac{K+1}{3K}K + K+1 > 0 \to -\dfrac{K+1}{3}+K+1>0 \to \dfrac{2K+2}{3}>0 \to K+1>0 \to \boxed{K>-1\ (II)}$

هیچ مقدار صحیح و منفی نیست

(سوالات تشریحی)

۸) ۱

$y' = 3x^2 + 2ax + b \xrightarrow{x=-1} y' = 3 - 2a + b = 0$

$\xrightarrow{x=-1} y = -1 + a - b - 1 = -2 + a - b = -4$

$\begin{cases} -2a + b = -3 \\ a - b = -2 \end{cases} \Rightarrow a = 5 , b = 7 \Rightarrow \frac{a}{b} = \frac{5}{7}$

عطف $x = \frac{-b}{3a} \to x = \frac{-a}{3} \to \frac{-a}{3} = -1 \to \boxed{a = 3}$

$-4 = -1 + 3 - b - 1 \to \boxed{b = 5}$

$\frac{a}{b} = \frac{3}{5}$

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۹)

$\left.\begin{array}{l} f'(x) = 3x^2 + 2ax + b \\ 0 = f'(x) \leftarrow x = 0 \end{array}\right\} b = 0$

$f'(x) = x(3x + 2a)$ → $-\frac{2a}{3}$

$\underset{0}{f(x)} = \left(\frac{-2a}{3}\right)^3 + a\left(-\frac{2a}{3}\right)^2 + 4 \longrightarrow a = -3$

$\frac{-2(-3)}{3} = 2$ ✓ ۲

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۱۰)

$f'(x) = 4x^3 - 12x \longrightarrow 4x(x^2 - 3)$

$\longrightarrow f'' = 12x^2 - 12 \longrightarrow x = \pm 1$ نقاط عطف

$-\sqrt{3}$		0		$\sqrt{3}$	
$-$		$+$	$-$		$+$
↘		↗	↘		↗

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$A(\sqrt{3}, -4) \quad B(-\sqrt{3}, -4) \quad m_{AB} = 0$

$C(1, 0) \quad D(-1, 0)$

خطوط یکدیگر را قطع X در نقاط ~~چند~~ تقاطع روی هم بیاندازیم.