تکلیف کنکوری — تکلیف ۲۷ — دوازدهم دفتر A

۱-

$f(x) = \cos^3 2x = 3\cos^2 2x \times 2 \times -\sin 2x + 2ax$

$x=0 \to 1 + 0 + b = 0 \Rightarrow b = -1$

$\lim \frac{f(x)}{x} = 0 \xrightarrow{0/0\ \text{Hop}} \frac{f'(x)}{1} \therefore f'(0^+) = 0 \to 6 \times 0 + 0 = 0$

$\lim \frac{f'(x)}{x} = 2 \xrightarrow{\text{Hop}} \frac{f''(x)}{1} = 2 \to f'(x) = -6\cos^2 2x \sin 2x + 2ax$

$f'(x) = -3\cos 4x \sin 4x + 2ax$

$f''(x) = -3(-2\sin x \sin 4x + 4\cos x \cos 2x) + 2a$

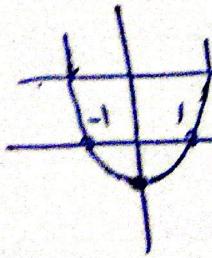
$f''(0) = -3\,(0 + 4) + 2a \Rightarrow 2a - 12 = 2$

$2a = 14$

$a = 7$

$a + b = 6$ ($a = 7$, $b = -1$)

۲-



$(\alpha, \alpha^2 - 1)$

$(\beta, \beta^2 - 1)$

$y = x^2 - 1$

$y' = 2x \to 2\alpha \times 2\beta = -1$

$4\alpha\beta = -1$

$\alpha\beta = -\frac{1}{4}$

$\alpha^2 + \beta^2 - 2 = ?$

$\alpha = \frac{1}{2}$

$\beta = -\frac{1}{2}$

$\frac{1}{4} + \frac{1}{4} - 2 = \frac{1}{2} - 2 = -1/5$

$\frac{1}{2}\times\frac{1}{2}-1=\frac{1}{2}-1=-\frac{1}{2}$

$\alpha=\frac{1}{2}$

$\beta=-\frac{1}{2}$

$(-0.5,-12)$
$(2.5,0)$
$\Rightarrow f'(x)=\frac{0-(-12)}{5}=\frac{12}{5}$

$y-0=\frac{12}{5}(x-2.5)$

$y=\frac{12}{5}x-6$

$f(x)=\frac{a}{2x-1}\Rightarrow f'(x)=\frac{0-2a}{(2x-1)^2}\Rightarrow\frac{-2a}{(2x-1)^2}=\frac{12}{5}$

$\frac{a}{2x-1}=\frac{12}{5}x-6$

$f(a)=\frac{a}{9}=\frac{-6}{9}=-\frac{1}{3}$

$12x^2-24x+9-a=0$

$\Delta=0\Rightarrow 576-144+48a=0$

$a=-3$

$y=\frac{x+a}{ax+1}\rightarrow y'=\frac{ax+1-ax-a^2}{(ax+1)^2}=\frac{1-a^2}{(ax+1)^2}$ $x=1\Rightarrow$

$\frac{1-a^2}{(a+1+a)^2}=2$ $\quad 2a^2+4a+2=1-a^2$

$3a^2+4a+1=0\rightarrow a^2+\frac{4}{3}a+\frac{1}{3}=0$

$(a+1)(a+\frac{1}{3})$

$a=-1\Rightarrow y=0\Rightarrow y=\frac{-1}{2}x+b$ $\quad a-b=-1-2=-3$ $\quad -1,\ -\frac{1}{3}$

$b=2$

$x=-\frac{1}{3}\rightarrow y=\frac{\frac{2}{3}}{\frac{2}{3}}=1\Rightarrow 2+b=1$

$b=-1$

$a-b=-\frac{1}{3}+1=\frac{2}{3}$

$\sin x + \frac{1}{2}\cos x = \frac{\sqrt{2}}{2}\sin x$

$\frac{1}{2}\cos x = \frac{1}{2}\sin x \Rightarrow \sin x = \cos x$

$\Rightarrow x = \frac{\pi}{4}$, $\frac{5\pi}{4}$

$[0, \pi]$ في $\rightarrow x = \frac{\pi}{4}$

$f'(x) = \cos x + \frac{1}{2}(-\sin x) = \frac{\sqrt{2}}{2} + \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right) = \frac{1}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}$

$f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{2\sqrt{2}+\sqrt{2}}{4} = \frac{3\sqrt{2}}{4}$

$f(x) = 2x^3 - 6x^2 - 12x + 1 \rightarrow f'(x) = 6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x-2=0$, $x=2$ (1) (-1) $x+1=0$, $x=-1$

$(2, -19)$

$(-1, 8)$

$\rightarrow f'(x) = \frac{8 - (-19)}{-1-2} = \frac{27}{-3} = -9$

$6x^2 - 6x - 12 = -9$

$6x^2 - 6x - 3 = 0 \Rightarrow 2x^2 - 2x - 1 = 0$

$x^2 - 2x - 1 = 0$

$\Delta = 4 - 4 \times 1 \times -2 > 0$ وتقطع

$y = 0$

$-4\sqrt{2}x + \sqrt{2}\pi + 12\sqrt{2} = 0$

$-4x + \pi + 12 = 0$

$\pi + 12 = 4x$

$x = 1 + \frac{\pi}{4}$

$y - 1^2\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}(x - \frac{\pi}{4})$

$y = -\sqrt{2}x + \frac{\sqrt{2}\pi}{4} + 12\sqrt{2}$

$6x^2-6x-12=-9$

$6x^2-6x-3=0 \Rightarrow 2x^2-2x-1=0$

$x^2-2x-2=0$

$\Delta = 4-4\times1\times(-2)>0$ دو نقطه

۷-

$y = kx^3+(k+1)x^2 = x^2(kx+k+1)$

$\rightarrow kx+k+1=0$

$kx=-k-1$

$x=\dfrac{-k-1}{k}\geq 0$

$y' = 3kx^2+(2k+2)x$

$y'' = 6kx+2k+2=0$

$6kx=-2k-2$

$x=\dfrac{-2k-2}{6k}\leq 0$

-1		0	
$-$	$+$		$-$

$D_g=[-1,0]$

-1		0	
$-$	$+$		$-$

$D_f=[0,+\infty)\cup(-\infty,-1]$

$\rightarrow k=-1$ مقدار

$D_f\cap D_g=\{-1,0\}$ منفی

۸-

$y=x^3+ax^2+bx-1 \Rightarrow y'=3x^2+2ax+b$

$-1+a-b-1$

$-2+a-b=-4$

$f'(-1)=3-2a+b=0$

$-2a+b=-3$

$a-b=-2$

$-a=-5 \rightarrow a=5$

$b=7$

$\dfrac{a}{b}=\dfrac{5}{7}$

۹ -

$f(x) = x^3 + ax^2 + bx + c \Rightarrow c = 4$

$\Rightarrow f'(x) = 3x^2 + 2ax + b \Rightarrow f'(0) = 0 + b = 0 \Rightarrow b = 0$

$= x(3x + 2a) = 0 \Rightarrow 3x = -2a \Rightarrow x = -\frac{2a}{3}$

$x^3 + ax^2 + 4 \Rightarrow -\frac{8a^3}{27} + \frac{4a^3}{9} + 4 = 0 \Rightarrow \frac{-8a^3 + 12a^3}{27} = -4$

$\frac{4a^3}{27} = -4 \Rightarrow a^3 = -27$; $a = -3$ $\rightarrow x = -\frac{2a}{3} = \frac{-2 \times -3}{3} = 2$

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۱۰ -

$f(x) = x^4 - 6x^2 + 5 \rightarrow f'(x) = 4x^3 - 12x = 0$

$4x(x^2 - 3) = 0$

$f''(x) = 12x^2 - 12 = 0$

$x^2 = 1$

$x = 1$

$x = -1$

$(1, 0)$

$(-1, 0)$

$-\sqrt{3}$		0		$\sqrt{3}$	
$-$		$+$	$-$		$+$
Min نسبی		Max نسبی		Min نسبی	

$A(-\sqrt{3}, -4)$

$B(\sqrt{3}, -4)$

زاویه ۹۰° باهم می‌سازند