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تمرین ۲۷: بدون مشتق / یکی دهمانی / دوازدهم دهتر A

$f(x)=\cos^2(\pi x)+ax^2+b \;,\; f'(x)=-2\times\pi\times\sin(\pi x)\cos(\pi x)+2ax$ (۱)

$\lim_{x\to 0^+}\frac{f(x)}{x}=\lim_{x\to 0^+}\frac{\cos^2\pi x+ax^2+b}{x}$ هم‌ارزی اولیه $\lim_{x\to 0^+}\frac{1-\pi^2x^2+ax^2+b}{x}=0$

چون حد حاصل داریم پس باید صورت کسر وقتی $x\to 0^+$ صفر شود پس $b=0$

$\lim_{x\to 0^-}\frac{f'(x)}{x}=\lim_{x\to 0^-}\frac{-2\sin(\pi x)\cos(\pi x)+2ax}{x}$ هم‌ارزی اولیه $\lim_{x\to 0^-}\frac{-2\pi x+2a}{x}=\lim_{x\to 0^-}-2\pi+2a$

$\to \lim_{x\to 0^-}-2\pi x+2a=2 \to a=1 \Rightarrow a+b=1+0=$ (۱) ✓

$y=x^2 \to f'(x)=2x \Rightarrow -2x^2=-1 \to x=\pm\frac{1}{2}$ (۲)

$f'(-x)=-2x$ ✗ حاصل ضرب شیب‌ها خط عمود = ۱-

$\begin{cases} x=\frac{1}{2} \to \frac{1}{2}-1=-\frac{1}{2}=y_1 \\ x=-\frac{1}{2} \to \frac{1}{2}-1=-\frac{1}{2}=y_2 \end{cases} \to y_1+y_2=\frac{-2}{2}=\frac{-3}{3}$ ✓

$m_{\text{شیب}}=\frac{\Delta y}{\Delta x}=\frac{4-(-12)}{2\sqrt{5}-(-\sqrt{5})}=\frac{16}{3\sqrt{5}}=4 \to y=4x+b$ (۳)

$(\sqrt{5},4) \to y=4\sqrt{5}+b \to b=-9 \Rightarrow y=4x-9$

$f(x)=\frac{a}{2x-1}=4x-9 \to a=8x^2-18x+9$

$f'(x)=\frac{-2a}{(2x-1)^2}=4 \to a=-2(4x^2-4x+1)=-8x^2+8x-2$

$\to -8x^2+8x-2=8x^2-18x+9 \to 16x^2-26x+11=0$

$\to 2x^2-3x+1=0 \to (x-1)(2x-1)=0 \to x=\frac{1}{2}$ یا $x=1$ ق ق

$f(x)=$ ت ق

(ادامه در پست بعد!)

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$f(1) = 4 - 9 = \frac{a}{2-1} \Rightarrow a = -5$

$f(0) = \frac{-5}{1-2} = \frac{-1}{2}$ ✓

$f'(x) \to \frac{ax+1-ax-a^2}{(ax+1)^2} \to f'(1) = \frac{1-a^2}{(a+1)^2} = \frac{1-a}{1+a}$ (۳)

$g'(x) \to 2 \Rightarrow g'(1) = f'(1) \Rightarrow \frac{1-a}{1+a} = 2$

$\Rightarrow 2+2a = 1-a \to \boxed{a = -\frac{1}{3}}$

$f(1) = g(1) \Rightarrow \frac{a+1}{a+1} = 2+b \Rightarrow 2+b = 1 \to b = -1$

$\Rightarrow a+b = -\frac{1}{3} + 1 = \frac{2}{3}$ ✓

$f(x) = g(x) \to \frac{\sqrt{2}}{2}\sin x = \sin x + \frac{1}{2}\cos x$ (۵)

$\to \frac{1}{2}\sin x = \frac{1}{2}\cos x \to \sin x = \cos x \xrightarrow{x \in [0,\pi]} x = \frac{\pi}{4}$

$f'(x) = \cos x - \frac{1}{2}\sin x \to f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4}$ شیب

$f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4} = \frac{3\sqrt{2}}{4} \Rightarrow y = \frac{\sqrt{2}}{4}x + b$

$x = \frac{\pi}{4} \to \frac{\sqrt{2}}{4} \times \frac{\pi}{4} + b = \frac{3\sqrt{2}}{4} \to b = -\frac{\pi\sqrt{2}}{16} + \frac{3\sqrt{2}}{4}$

$y = 0 \to \frac{\sqrt{2}}{4}x + \frac{3\sqrt{2}}{4} - \frac{\pi\sqrt{2}}{16} = 0 \to \boxed{x = \frac{\pi}{4} + 3}$ ✓

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(۲) طرفین بهره‌ی (۱) ← شیب = ۰ ⇐ متن نامفهوم باشد

$f'(x) = 4x^2 - 4x - 12 = 0 \to x^2 - x - 3 = 0 \to x = -1 \text{ و } x = 3$

$f(-1) = -2 - 4 + 12 + 1 = 8$ و $f(3) = 16 - 12 - 24 + 1 = -19$

شیب $m_{AB} = \frac{\Delta y}{\Delta x} = \frac{-19-8}{3+1} = \frac{-27}{4} = -9$

$f'(x) = 9 \to 4x^2 - 4x - 12 = -9 \to 4x^2 - 4x - 3 = 0$

$\Delta > 0 \to$ دو ریشه $\to$ [illegible]

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$y' = 3kx^2 + 2(k+1)x \rightarrow y'' = 6kx + 2(k+1) = 0$ (۷)

$\rightarrow x = \frac{-k-1}{3k}$ نقطه درنامیدرک طول ⊖، عرض ⊕ است.

(I) $x<0 \rightarrow \frac{-k-1}{3k} < 0 \rightarrow$ (- | + | -) $\rightarrow x \in (-\infty, -1) \cup (0, +\infty)$

(II) $y_2 \; kx\left(\frac{-k-1}{3k}\right)^2 + (k+1)\left(\frac{-k-1}{3k}\right)^2 > 0 \rightarrow \frac{-(k+1)^2}{9k^2} + \frac{-(k+1)^2}{27k^2}$

$= \frac{-4(k+1)^2}{27k^2} > 0 \rightarrow$ (+ | ∄ | -) $\rightarrow x \in (-\infty, -1)$

(I) ∩ (II) $\Rightarrow x \in (-\infty, -1) \rightarrow$ بزرگ!

$y' = 3x^2 + 2ax + b \xrightarrow{x=-1} 3 - 2a + b = m$ شیب (۱)

$y_2 \; x^3 + ax^2 + bx - 1 \xrightarrow{x=-1} -1 + a - b - 1 = -4 \rightarrow a - b = -2$

نقطه فقط یک خط مماس می‌تواند از دون منفی عبور کند ⟵

$\hookrightarrow m=0 \rightarrow 3 - 2a + b = 0 \rightarrow 2a - b = 3$

$\begin{cases} 2a - b = 3 \\ -a + b = +2 \end{cases} \Rightarrow a = 5, \; b = 7 \Rightarrow \frac{a}{b} = \frac{5}{7} \rightarrow$ (۵)

$f(x) \Rightarrow x = 0 \rightarrow \boxed{c = 4}$ (۹)

$f'(x) = 4x^3 + 2ax + b = 0 \xrightarrow{x=0} \boxed{b = 0}$

$\rightarrow f'(x) = 4x^3 + 2ax = 0 \rightarrow x(4x + 2a) = 0 \rightarrow x = 0, \; x = \frac{-2a}{4}$

$x = \frac{-2a}{4} \rightarrow f\left(\frac{-2a}{4}\right) = \frac{-16a^4}{256} + \frac{16a^4}{256} + 4 = 0 \rightarrow \frac{64a^4}{256} = 4$

$a^4 = -256 \rightarrow \boxed{a = -4}$

$\min \Rightarrow x = \frac{-2a}{4} = \frac{-2 \times -4}{4} = 2 \rightarrow$ (۲)

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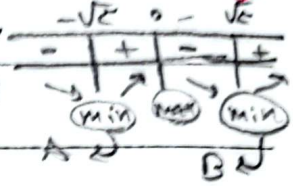
(۱۵) * نقاط بحرانی $\Rightarrow$ مشتق اول $=0$ و * نقاط عطف $\Leftarrow$ مشتق دوم $=0$

$f'(x) = 4x^3 - 12x = 0 \rightarrow x = \pm\sqrt{3}, 0 \rightarrow$



$f''(x) = 12x^2 - 12 = 0 \rightarrow x = \pm 1$

$A: x = -\sqrt{3} \rightarrow f(-\sqrt{3}) = 9 - 18 + 5 = -4 \rightarrow A(-\sqrt{3}, -4)$

$B: x = \sqrt{3} \rightarrow f(\sqrt{3}) = 9 - 18 + 5 = -4 \rightarrow B(\sqrt{3}, -4)$

$C: x = 1 \rightarrow 1 - 6 + 5 = 0 \rightarrow C(1, 0)$

$D: x = -1 \rightarrow 1 - 6 + 5 = 0 \rightarrow D(-1, 0)$

* AB و CD پاره خط‌هایی هستند که $\frac{\Delta y}{\Delta x} = 0$ است پس زاویه‌ی شان $\theta = 0$ است