1

$$\lim_{x\to 0^-} \frac{f'(x)}{x} = 2$$

$f(x) = \cos^3(2x) + ax^2 + b \rightarrow f'(x) = 3\times 2(-\sin 2x)\times \cos^2 2x + 2ax$

$$\frac{-6\sin 2x\cos^2 2x + 2ax}{x} \xrightarrow{\text{hop}} = \frac{-6(2\cos 2x\times\cos^2 2x + \sin 2x\times 2\cos 2x\times(-2\sin 2x)) + 2a}{1} = 2$$

$-6\times 2 + 2a = 2 \rightarrow a = 7$

$$\lim_{x\to 0} \frac{f(x)}{x} = 0 \rightarrow \frac{\cos^3(2x) + ax^2 + b}{x} \rightarrow$$ باید صورت و مخرج صفر شود $\rightarrow b = -1$

$a + b = 6$

2

$y = x^2 - 1 \rightarrow y' = 2x$

$2x(-2x) = -1 \rightarrow x = \pm\frac{1}{2} \rightarrow y = \frac{1}{4} - 1 = -\frac{3}{4}$

$-4x^2$

$-\frac{3}{4}\times 2 = -\frac{3}{2}$

3

$m = \frac{6+12}{2/5+0/5} = 6$

$\rightarrow 6x - 9 = y$

$f(x) = \frac{a}{2x-1} \rightarrow f'(x) = \frac{-a\times 2}{(2x-1)^2} = 6$ 3

$\rightarrow 6x - 9 = \frac{a}{2x-1}$

$a = -3(2x-1)^2 = (6x-9)(2x-1)$

$2x - 3$

$-4x^2 + 4x - 1 = 4x^2 - 8x + 3 \rightarrow 8x^2 - 12x + 4 = 0$ ($2x^2 - 3x + 1 = 0$) $\quad x = 1$ و $\frac{1}{2}$ $\rightarrow a = -3$

$\rightarrow f(5) = \frac{-3}{10-1} = -\frac{1}{3}$

4

$y = \frac{x+a}{ax+1}$ ($\rightarrow 1+a$, $\rightarrow 1+a$)

$\rightarrow \frac{(1+a) - a(1+a)}{(1+a)^2} = 2$ ($1-a^2$) $\rightarrow 2a^2 + 4a + 2 = 1 - a^2$

$3a^2 + 4a + 1 = 0 \quad a = -1$ و $-\frac{1}{3}$

$y = \frac{x - \frac{1}{3}}{-\frac{1}{3}x + 1} = \frac{\frac{2}{3}}{\frac{2}{3}} = 1 \xrightarrow{(1,1)} y = 2x + b \rightarrow 1 = 2 + b \rightarrow b = -1$

$a - b = \frac{2}{3}$

$-\frac{1}{3} + 1$

$g(x) = \frac{3}{2}\sin x \qquad f(x) = \sin x + \frac{1}{2}\cos x$

$\frac{3}{2}\sin x = \sin x + \frac{1}{2}\cos x \rightarrow \sin x = \cos x \xrightarrow[x\in[0,\pi]]{} x = \frac{\pi}{4} \rightarrow f(x) = \frac{3\sqrt{2}}{4}$

$\frac{1}{2}\sin x$

$\rightarrow f'(x) = \cos x - \frac{1}{2}\sin x \xrightarrow[x=\frac{\pi}{4}]{} \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4}$

$y = \frac{\sqrt{2}}{4}x + b \qquad \frac{3\sqrt{2}}{4} \qquad \frac{\pi}{4} \qquad -\frac{\pi\sqrt{2}}{16} + \frac{3\sqrt{2}}{4}$

خواسته سوال $\rightarrow \frac{\sqrt{2}}{4}x = \frac{\pi\sqrt{2}}{-16} - \frac{3\sqrt{2}}{4} \rightarrow \boxed{x = \frac{\pi}{4} - 3}$

$f(x) = 2x^3 - 3x^2 - 12x + 1 \xrightarrow{f'(x)} 6x^2 - 6x - 12 \rightarrow 6(x-2)(x+1)$

$A = (2, -19) \quad B = (-1, 8) \rightarrow m_{AB} = \frac{-27}{3} = -9$

$\rightarrow 6x^2 - 6x - 12 = -9 \rightarrow 2x^2 - 2x - 1 \xrightarrow[\Delta > 0]{}$ ۲ نقطه وجود دارد *

(−3)

$y = kx^3 + (k+1)x^2 \rightarrow y' = 3kx^2 + 2(k+1)x \rightarrow y'' = 6kx + 2(k+1)$

$0 = 6kx + 2k + 2 \rightarrow x = \frac{-2k-2}{6k}$ نقطه عطف

$\frac{-2k-2}{6k} < 0$ ناحیه ۲

-1		0	
−	+		−

$k \in (-\infty, -1) \cup (0, +\infty)$

$\left(\frac{-2k-2}{6k}\right)^2 \left(\frac{-2k-2}{6} + k + 1\right) > 0$

$0 \qquad 3 \qquad \frac{3k+3}{3}$ ناحیه ۲

-1	
−	+

$-1 \leftarrow \frac{2k+2}{3}$

$\cap = \varnothing$

به ازای هیچ مقدار صحیح و غیر منفی k

$y = x^3 + ax^2 + bx - 1 \rightarrow -1 + a - b - 1 = -4$

$(-1, -4)$

-2

$\rightarrow y' = 3x^2 + 2ax + b \xrightarrow[x=-1]{} 3 - 2a + b = 0$

$\begin{cases} a - b = -2 \\ -2a + b = -3 \end{cases}$

$-a = -5 \rightarrow a = 5$

$b = 7$

$\frac{a}{b} = \frac{5}{7}$

9

$f(x) = x^{3} + ax^{2} + bx + c \to f'(x) = 3x^{2} + 2ax + b$ } $\to b = 0$

(b = ۰ ، c = ۴)

در $x = 0 \to f'(x) = 0$

$\to f'(x) = x(3x + 2a)$ → $\frac{-2a}{3}$

$f(x) = \left(\frac{-2a}{3}\right)^{3} + a\left(\frac{-2a}{3}\right)^{2} + 4$ ($f(x) = 0$, $4 \to -4$)

خواسته سوال

$\frac{-8a^{3}}{27} + \frac{4a^{3} \times 3}{3 \times 9} = -4$ ($\frac{4a^{3}}{27}$) $\to a = -3 \to \frac{-2a}{3} = \frac{-2(-3)}{3} = 2$

10

$f(x) = x^{4} - 6x^{2} + 5 \to f'(x) = 4x^{3} - 12x \to 4x(x^{2} - 3)$

$\to f''(x) = 12x^{2} - 12 \to x = \pm 1$ نقاط عطف D و C

B و A: min نسبی

$-\sqrt{3}$	0	$\sqrt{3}$	
−	+	−	+
↘	↗	↘	↗

$A(\sqrt{3}, -4)$ $B(-\sqrt{3}, -4) \to m_{AB} = 0$

$C(1, 0)$ $D(-1, 0)$

* خطوط شیب‌ها را قطع نمی‌کنند
زاویه صفر درجه اگر روی هم بیاندازیم شان