نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۷ ... کلاس دانش‌آموزی A .....

۱۴۰۵

۱

$\cos^2 2x + ax^2 + b \longrightarrow -2\times 2\cos 2x \sin 2x + 2ax \longrightarrow 2a - 4\times 2\times(-\sin 2x)\times\cos 2x + 2\sin 2x$

مشتق مرتب هست

$+ 2\cos 2x$

$2+2a=2$ $\boxed{a=0}$ $\boxed{b=0}$ $\rightarrow \boxed{a+b=0}$ جواب

$\lim \frac{f'(x)}{x}$ و وقتی $x\to 0$ $\Rightarrow f'(0)$ یعنی $\lim \frac{f(x)}{x} = 0$

$f''(0) = 2$ است.

(جواب صفر بود)

۲

$f'(x) = 2x$

$2x^2 = 1 \rightarrow x = \pm\frac{1}{2}$ $\rightarrow -\frac{3}{4}$, $-\frac{3}{4}$ $\Rightarrow$ کمترین $\left(-\frac{3}{4}\right)$ جواب ✓ (۲)

۳

$\frac{a}{2a-1} = (4a-9) \rightarrow 16a^2 - 8a - 4a + 9 - a = 0 \rightarrow 16a^2 - 24a + 9 - a = 0$

$f(a) = \frac{-1}{3}$

$\Delta = 0 \rightarrow (24)^2 - 4(16)(9-a) = 0 \rightarrow 576 - 432 + 64a = 0 \rightarrow 64a = -144 \rightarrow a = -\frac{9}{4}$

۳

$f(a) = \frac{-1}{3}$ (۱)

$f' = \frac{0 - (4x-1) - 2a}{(2x-1)^2} = \frac{-2a}{(2x-1)^2}$ $\Delta = 0$ حتما برای صفر شدن است

$y = \frac{-24}{2x-1} = \frac{-24}{9} = \frac{-8}{3}$ جواب

$\frac{-2a}{(2x-1)^2} = 12$ $\rightarrow a = -24$

$y = 4x + 9$

$2x - 9 = \frac{-24}{(2x-1)^2}$

$m = \frac{-42 - (-4)}{-10 - 2/5} = \frac{-18}{-40} = \frac{9}{20}$ (۴)

۴

مشتق $\rightarrow f' = 2$

$\frac{1(ax+1) - (x+a)a}{(ax+1)^2} = \frac{1-a^2}{(ax+1)^2} = \frac{1-a^2}{(a+1)^2}$

$2 = \frac{1-a^2}{a^2+1+2a}$ $2a^2 + 2 + 4a = 1 - a^2$ (۲)

$3a^2 + 1 + 4a = 0 \rightarrow a = -\frac{1}{3}$ ✓ $a - b = \frac{1}{2}$ جواب (۲/۵)

$x = 1 \rightarrow \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3} = y$ $2 + b = 1$ $\boxed{b = -1}$

۵

$\frac{\sqrt{2}}{2}\sin x = \sin x - \frac{\sqrt{2}}{2}\cos x$ $\frac{\sqrt{2}}{2}\sin x = \frac{\sqrt{2}}{2}\cos x \rightarrow x = \frac{\pi}{4}$ و $\frac{5\pi}{4}$

$\frac{3\sqrt{2}}{2} = 3\frac{\sqrt{2}}{2}$

مقدار $\rightarrow \cos x - \frac{1}{2}\sin x = \cos\frac{\pi}{4} - \frac{1}{2}\sin\frac{\sqrt{2}\pi}{4} = +\frac{\sqrt{2}}{4}$

$y = \frac{\sqrt{2}}{2}x + \frac{3\sqrt{2}}{4} - \frac{\sqrt{2}\pi}{8} \rightarrow \boxed{x = \frac{3\sqrt{2} - \frac{\sqrt{2}\pi}{8}}{\sqrt{2}/2}}$ جواب ✓ (۲)

۶

$y = x^3 - 3x^2 + 1$ ①

$y' = 3x^2 - 6x$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0, \quad x = 2$

$y(0) = 1, \quad y(2) = -3$

$\begin{array}{c|cc} x & -\infty & 0 & 2 & +\infty \\ \hline y' & + & 0 & - & 0 & + \\ y & \nearrow & \text{max} & \searrow & \text{min} & \nearrow \end{array}$

$y = -2x + 1$ ②

$m = -2$

$\begin{vmatrix} 2 & -1 \\ -19 & 8 \end{vmatrix} = 16 - 19 = -3$

$\begin{vmatrix} -1 & -1 \\ 8 & -3 \end{vmatrix} = 3 + 8 = 11$

$\frac{11}{-3} = -\frac{11}{3}$

$y = -\frac{11}{3}x + 9$

$3y = -11x + 27$

$11x + 3y - 27 = 0$

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$y = x^3 - 3x^2 + 2x + 1$

$y' = 3x^2 - 6x + 2$

$y'' = 6x - 6$

$y'' = 0 \Rightarrow x = 1$

$y(1) = 1 - 3 + 2 + 1 = 1$

نقطه عطف $(1, 1)$

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$y = x^3 + ax^2 + bx$

$y' = 3x^2 + 2ax + b$

$y'' = 6x + 2a$

$y'' = 0 \Rightarrow x = -\frac{2a}{6} = -\frac{a}{3}$

$-\frac{a}{3} = -1 \Rightarrow a = 3$

$y' = 3x^2 + 6x + b$

$y'(-1) = 0$

$3 - 6 + b = 0 \Rightarrow b = 3$

$\frac{a}{b} = \frac{3}{3} = 1$

۹

$y = x^3 + ax^2 + bx + c$

$y' = 3x^2 + 2ax + b$

$y'(0) = 0 \Rightarrow b = 0$

$y = x^3 + ax^2 + c$

$y(-2) = 4$

$-8 + 4a + c = 4$

$4a + c = 12$

$y'(-2) = 0$

$12 - 4a = 0 \Rightarrow a = 3$

$c = 0$

$y = x^3 + 3x^2$

$y' = 3x^2 + 6x$

$y'' = 6x + 6$

$y'' = 0 \Rightarrow x = -1$

$y(-1) = -1 + 3 = 2$

نقطه عطف $(-1, 2)$

۱۰

$y = x^3 - 3x$

$y' = 3x^2 - 3$

$y' = 0 \Rightarrow x = \pm 1$

$y(1) = -2, \quad y(-1) = 2$

$A(1, -2), \quad B(-1, 2)$

$m_{AB} = \frac{2 - (-2)}{-1 - 1} = \frac{4}{-2} = -2$

$y = -2x + b$

$2 = -2(-1) + b \Rightarrow b = 0$

$y = -2x$

$2x + y = 0$

سؤال ۱

$f(x)=\cos^{3}(2x)+ax^{2}+b$

$\lim_{x\to 0^{+}}\frac{f(x)}{x}=0 \rightarrow \lim_{x\to 0^{+}}\frac{\cos^{3}(2x)+ax^{2}+b}{x}=0 \rightarrow \lim_{x\to 0^{+}}\frac{1+b}{x}=0 \rightarrow \boxed{b=-1}$

$\lim_{x\to 0^{-}}\frac{f'(x)}{x}=2 \rightarrow \lim_{x\to 0^{-}}\frac{-6\sin(2x)\cos^{2}(2x)+2ax}{x}=2 \xrightarrow{\text{sin}} \lim_{x\to 0^{-}}\frac{-6\times 2x+2ax}{x}=2$

$\rightarrow \lim_{x\to 0^{-}}\frac{(2a-12)x}{x}=2 \rightarrow 2a-12=2 \rightarrow \boxed{a=7}$

$a+b=6$

سؤال ۲

$f(1)=2 \rightarrow (1)^{3}+a(1)+b(1)+c=2 \rightarrow \boxed{c=2}$

$f'(x)=3x^{2}+2ax+b \rightarrow f'(1)=0 \rightarrow f'(1)=3(1)^{2}+2a(1)+b=0 \rightarrow \boxed{b=0}$

$f(x)=x^{3}+ax^{2}+2 \rightarrow f'(x)=3x^{2}+2ax=0 \rightarrow x(3x+2a)=0 \rightarrow \begin{cases} x=0 \\ x=-\frac{2a}{3} \end{cases}$

x		0		$-\frac{2a}{3}$	
y'	$+$		$-$		$+$
y	$\nearrow$		$\searrow$	min	$\nearrow$

$\rightarrow f(-\frac{2a}{3})=0 \rightarrow (-\frac{2a}{3})^{3}+a(-\frac{2a}{3})^{2}+2=0$

$\rightarrow \frac{-8a^{3}}{27}+\frac{4a^{3}}{9}+2=0 \rightarrow a^{3}=-27 \rightarrow \boxed{a=-3}$

$x=-\frac{2a}{3} \rightarrow \boxed{x_{min}=\frac{-2(-3)}{3}=2}$