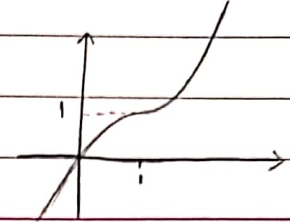


سوال ۱) $y = x^3 - 3x^2 + 3x$

الف) $y' = 3x^2 - 6x + 3 \rightarrow$ مشتق صفر یا تعریف نشده = نقاط بحرانی $\rightarrow y' = 0 \rightarrow 3x^2 - 6x + 3 = 0 \rightarrow 3(x^2 - 2x + 1) = 3(x-1)^2$
 $\rightarrow 3(x-1)^2 = 0 \rightarrow \boxed{x=1}$

ب) $y = x^3 - 3x^2 + 3x = x(x^2 - 3x + 3)$

y'	↑	↓	↑
y	-	+	+



الف) $y = \frac{-x^3 + 4}{x^2} \rightarrow y' = \frac{(-3x^2)(x^2) - (4x)(-2x^2 + 4)}{x^4} = \frac{-3x^4 + 8x^2 - 4x}{x^4} = \frac{-x^4 - 4x}{x^4}$

سوال ۲)

$\frac{-x(x^3 + 4)}{x^4} \rightarrow x=0$ تن
 $\rightarrow x = -2 \rightarrow 0$
 \rightarrow نقاط بحرانی $x = \{0, 2\}$

ب) $y = \frac{x^3}{x^2 - 1} \rightarrow y' = \frac{(3x^2)(x^2 - 1) - (4x)(x^3)}{(x^2 - 1)^2} = \frac{3x^4 - 3x^2 - 4x^4}{(x^2 - 1)^2} = \frac{-x^4 - 3x^2}{(x^2 - 1)^2} = \frac{-x^2(x^2 + 3)}{(x^2 - 1)^2} = 0$

$\rightarrow x = \pm 1$ تن
 $\rightarrow x = \pm\sqrt{3}, 0$
 \rightarrow نقاط بحرانی $x = \{-\sqrt{3}, -1, 0, 1, \sqrt{3}\}$

الف) $y = \frac{-x^3 + 4x + 1}{x - 1} \rightarrow y' = \frac{(-3x^2 + 4)(x - 1) - (1)(-x^3 + 4x + 1)}{(x - 1)^2} = \frac{-3x^3 + 3x^2 + 4x - 4 + x^3 - 4x - 1}{(x - 1)^2} = \frac{-2x^3 + 3x^2 - 5}{(x - 1)^2}$

سوال ۳)

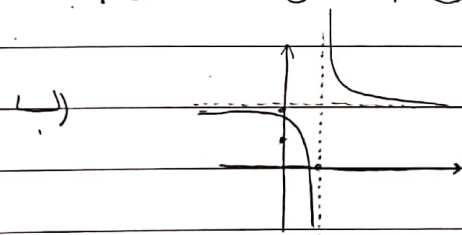
$= \frac{-2x^3 + 3x^2 - 5}{(x - 1)^2} \neq 0 \rightarrow$ مشتق ندارد \rightarrow ندارد ent

ب) $y = \frac{x^3 - 4x + 3}{x - 1} \rightarrow y' = \frac{(3x^2 - 4)(x - 1) - (1)(x^3 - 4x + 3)}{(x - 1)^2} = \frac{3x^3 - 3x^2 - 4x + 4 - x^3 + 4x - 3}{(x - 1)^2} = \frac{2x^3 - 3x^2 + 1}{(x - 1)^2} = 1$

$\rightarrow y' \neq 0 \rightarrow$ ندارد ent

الف) $\left\{ \begin{array}{l} \text{مجاذب قائم} = \text{مشتق خارج} \rightarrow x - 1 = 0 \rightarrow \boxed{x=1} \\ \text{مجاذب افقی} = \frac{a}{c} = \frac{3}{1} = \boxed{3} \end{array} \right.$

سوال ۴) $y = \frac{3x + 3}{x - 1}$



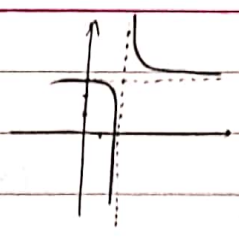
* از ناحیه ۳ عبور می کند

الف) $y = \frac{ax + f}{x - b} \rightarrow$ مرکز ثقل $(2, 3) \rightarrow b = 2, \frac{a}{1} = 3 \rightarrow a = 3$

سوال (5)

ب) $y = \frac{3x + f}{x - 2} \rightarrow y^{-1} = -\frac{-2x - f}{x - 3} = \frac{2x + f}{x - 3}$

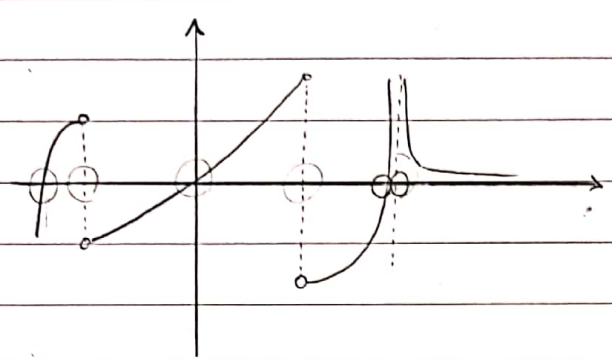
$y = \frac{3x + 1}{x - 2}$
 جانب افقی: $y = 3$
 جانب عمودی: $x = 2$



مرکز ثقل: $(2, 3)$

سوال (6)

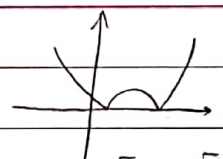
عوض $y = 1x + b \rightarrow 3 = 1 + b \rightarrow b = 2 \rightarrow y = x + 1$
 $y = -1x + b \rightarrow 3 = -2 + b \rightarrow b = 5 \rightarrow y = -x + 5$



عوض بحرانی دارد

سوال (7)

$y = |x^2 - ax + 2| \rightarrow$



در واقع باید پارابول باشد \rightarrow نقطه بحرانی دارد

سوال (8)

$\rightarrow \Delta > 0 \rightarrow a^2 - 4 > 0 \rightarrow \begin{matrix} -\sqrt{4} & \sqrt{4} \\ + & - & + \end{matrix} \rightarrow a : (-\infty, -2) \cup (2, +\infty)$

$y = \frac{x^2 + 2}{x^2 + x + 2} \rightarrow y' = \frac{(2x)(x^2 + x + 2) - (x^2 + 2)(2x + 1)}{(x^2 + x + 2)^2} \rightarrow \frac{2x^3 + 2x^2 + 4x - 2x^3 - 4x^2 - 4x - 2}{(x^2 + x + 2)^2}$

سوال (9)

$= \frac{x^2 - 2}{(x^2 + x + 2)^2} = 0 \rightarrow$

	$-\sqrt{2}$	$\sqrt{2}$	
y'	\nearrow	\searrow	\nearrow
y	+	+	+

$\rightarrow \max \begin{matrix} \sqrt{2} \\ \frac{4}{4 - \sqrt{2}} \end{matrix} \quad \min \begin{matrix} \sqrt{2} \\ \frac{4}{4 + \sqrt{2}} \end{matrix} \rightarrow \max, \min = \frac{14}{14 - \sqrt{2}} = \frac{14}{14} = 1$