آدرینا حسینی — کلاس حجد دوازدهم دفتر A — شماره تکلیف: ۳ — ۱۸

۱ (۲)

$y=\dfrac{x^2+x+2}{x-1} \to yx-y=x^2+x+2 \to x^2-x(1-y)+y+2=0 \to x_1,x_2=\dfrac{-b\pm\sqrt{\Delta}}{2a}$

$\Delta\geq 0 \to (1-y)^2-4(y+2)\geq 0 \to 1+y^2-2y-4y-8\geq 0 \to y^2-6y-7\geq 0 \to (y-7)(y+1)\geq 0$

$\begin{array}{ccccc} & -1 & & 7 & \\ + & 0 & - & 0 & + \end{array}$

$R_f=(-\infty,-1]\cup[7,+\infty)$

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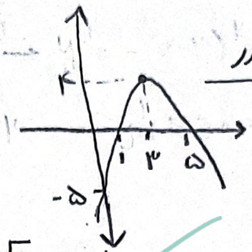
۲ (۲)

الف) $y=2x^2-5x+1 \to R_f=\mathbb{R}$

ب) $y=\sqrt{-x^2+6x-5}$

$\to \dfrac{-b}{2a}=\dfrac{-6}{-2}=3 \to y=-9+18-5=4$



$\xrightarrow{\text{جذر}} \sqrt{(-\infty,4]}=[0,2]$

ج) $y=\sqrt{x^7-3x^2+4x+1} \to \sqrt{\mathbb{R}} \to R_f=[0,+\infty)$

د) $\log(x^3-6x^2+7x+4) \to (-\infty,+\infty)$

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۳ (۱٫۷۵)

الف) $y=\dfrac{3x+1}{2x-6} \to R-\{\frac{3}{2}\}$

ب) $y=\sqrt{\dfrac{4x+1}{x-2}} \to \sqrt{R-\{2\}}=[0,+\infty)-\{2\}$

ج) $y=\dfrac{x^2+4}{\sqrt{x^2+3}} \to \underset{\text{اشتباه}}{\sqrt{x^2+3}}+\dfrac{1}{\sqrt{x^2+3}}=\sqrt{3}+\dfrac{1}{\sqrt{3}}=\dfrac{4}{\sqrt{3}}=\dfrac{4\sqrt{3}}{3} \to R_f=[\dfrac{4\sqrt{3}}{3},+\infty)$

د) $y=x^2+\dfrac{1}{x^2+4}+4-4 \to \underbrace{x^2+4}_{\min 4}+\dfrac{1}{x^2+4}-4 \to [-2,+\infty)$ — $[4.25,+\infty) \xrightarrow{-4} [\frac{1}{4},+\infty)$

0.25

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۴ (۱٫۲۵)

$y=\dfrac{3x^2+4}{x^2-1} \to yx^2-y=3x^2+4 \to x^2(y-3)=y+4 \to x^2=\dfrac{y+4}{y-3} \to x=\pm\sqrt{\dfrac{y+4}{y-3}}$

$\begin{array}{ccccc} & -4 & & 3 & \\ + & 0 & - & \nexists & + \end{array} \to R_f=(-\infty,-4]\cup(3,+\infty) \to x \begin{cases} +\infty & \dfrac{4}{-1}=-4 \\ 0 & \dfrac{4}{-1}=-4 \end{cases}$

$\dfrac{3}{1}=3$ مجانبی

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۵ (۲)

$y=\dfrac{2\sin x-1}{\sin x+3} \to y\sin x+3y=2\sin x-1 \to (y-2)\sin x=-3y-1 \to \sin x=\dfrac{-3y-1}{y-2}$

$\dfrac{-3y-1}{y-2}\geq -1 \to \dfrac{-3y-1+y-2}{y-2}\geq 0 \to \dfrac{-2y-3}{y-2}\geq 0$

$\begin{array}{ccccc} & -\frac{3}{2} & & 2 & \\ + & & - & & + \end{array}$

I $(-\infty,-\frac{3}{2}]\cup(2,+\infty)$

$\dfrac{-3y-1}{y-2}\leq 1 \to \dfrac{-3y-1-y+2}{y-2}\leq 0 \to \dfrac{-4y+1}{y-2}\leq 0 \to \begin{array}{ccccc} & \frac{1}{4} & & 2 & \\ - & 0 & + & \nexists & - \end{array} \to (-\infty,\frac{1}{4}]\cup(2,+\infty)$ II

$\text{I}\cap\text{II} \to R_f=[-\frac{3}{2},\frac{1}{4}]$

$\min \sin=-1 \to y=\dfrac{-3}{2}$

$\max \sin=1 \to y=\dfrac{1}{4}$

$\to R_f=[-\frac{3}{2},\frac{1}{4}]$

۶ — ۱٫۷۵

$f(x) = \dfrac{x^2 - x + \frac{1}{4}}{x^2 - x + 1} \rightarrow yx^2 - yx + y = x^2 - x + \frac{1}{4} \rightarrow x^2(y-1) - x(y-1) + y - \frac{1}{4} = 0$

$y \neq 1 \leftarrow$ ضریب $x^2 \neq 0$

$\rightarrow y_1, y_2 = \dfrac{-b \pm \sqrt{\Delta}}{2a} \rightarrow (y-1)^2 - 4(y - \frac{1}{4})(y-1) \geq 0 \rightarrow y^2 - 2y + 1 - 4y^2 + 5y - 1 \geq 0$

$\rightarrow -3y^2 + 3y \geq 0 \rightarrow -3y(y-1) \geq 0$

0		1
−	+	−

$D_f = \mathbb{R}$

$R_f = [0,1)$ $\leftarrow$ $\boxed{R_f = [0,1]}$

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۷ — ۲

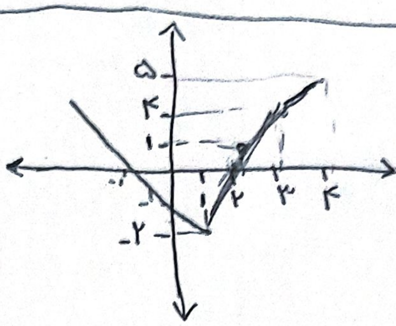
الف) $y = |2x-2| - |x-3|$

1		3
$-2x+2+x-3 \rightarrow -x-1$	$2x-2+x-3 \rightarrow 3x-5$	$2x-2-x+3 \rightarrow x+1$

1	0
−2	−1

2	3	1
1	4	−2

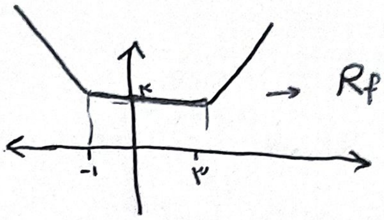
3	4
4	5



$\rightarrow R_f = [-2, +\infty)$

ب) $y = |\underbrace{x-3}_{3}| + |\underbrace{x+1}_{-1}|$

−1		3
$-x+3-x-1 \rightarrow -2x+2$	$-x+3+x+1 = 4$	$x-3+x+1 \rightarrow 2x-2$



$\rightarrow R_f = [4, +\infty)$

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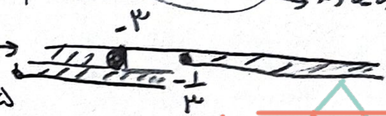
۸ — ۱

$|x-2| - |2x+2| \leq 1$

$x \in (-\infty, -3] \cup [-\frac{1}{3}, +\infty)$

−1		2
$-x+2+2x+2 \leq 1$, $x \leq -3$ (I)	$-x+2-2x-2 \leq 1$, $-3x \leq 1$, $x \geq -\frac{1}{3}$ (II)	$x-2-2x-2 \leq 1$, $-x \leq 5 \rightarrow x \geq -5$ (III)

$I \cap II \cap III \rightarrow$ $[-5, -3]$ → جواب نادرست



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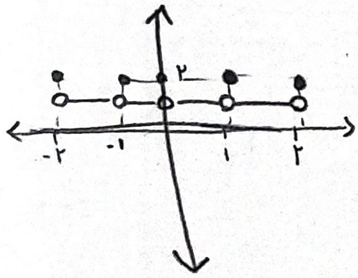
۹ — ۱٫۷۵

الف) $y = [x] + [-x] + 2$ $\begin{cases} x \in \mathbb{Z} \rightarrow 1 \\ x \notin \mathbb{Z} \rightarrow 2 \end{cases}$



ب) $y = 3x - [x]$

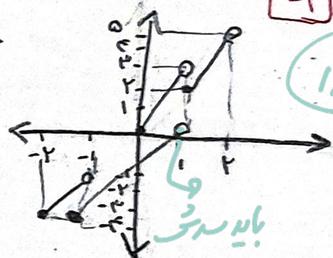
$-2 \leq x < -1 \xrightarrow{[x] = -2} 3x+2$ — $\begin{array}{c|c} -2 & -1 \\ \hline -4 & -1 \end{array}$

$-1 \leq x < 0 \xrightarrow{[x] = -1} 3x+1$ — $\begin{array}{c|c} -1 & 0 \\ \hline -2 & 1 \end{array}$

$0 \leq x < 1 \xrightarrow{[x] = 0} 3x$ — $\begin{array}{c|c} 0 & 1 \\ \hline 0 & 3 \end{array}$

$1 \leq x < 2 \xrightarrow{[x] = 1} 3x-1$ — $\begin{array}{c|c} 1 & 2 \\ \hline 2 & 5 \end{array}$

$x = 2 \rightarrow y = 4$



باید سرش روی $x=0$ باشد!

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۱۰ — ۲

الف) $y = \sin^2 x - \sin x + 1$

$1 \rightarrow 1$

$-1 \rightarrow 3$

$-\frac{b}{2a} \rightarrow (\frac{1}{2})^2 - \frac{2}{4} + \frac{4}{4} = \frac{3}{4}$

$\rightarrow R_f = [\frac{3}{4}, 3]$

ب) $y = \sin^4 x + \sin^2 x + 1$

$1 \rightarrow 3$

$0 \rightarrow 1$

$-\frac{b}{2a} \rightarrow (-\frac{1}{2})^4 + (-\frac{1}{2})^2 + 1 = \frac{1}{16} + \frac{4}{16} + \frac{16}{16} = \frac{21}{16} = 1\frac{5}{16}$

$\rightarrow R_f = [1, 3]$

$\sin^2 x \geq 0 \rightarrow \sin^2 x \neq -\frac{1}{2}$