

$$y = \frac{x^2 + x + 2}{x - 1} \rightarrow yx - y = x^2 + x + 2 \rightarrow x^2 + (1 - y)x + 2 + y = 0 \rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$x = \frac{-1 + y \pm \sqrt{(1 - y)^2 - 4(2 + y)}}{2} \rightarrow (1 - y)^2 - (4 + 4y) \geq 0$$

$$y^2 - 4y - 4 \geq 0 \rightarrow (y + 1)(y - 5) \geq 0$$

$$\frac{-1}{+} \frac{5}{-} \rightarrow R = (-\infty, -1] \cup [5, +\infty)$$

الف) $y = x^2 - 2x + 1 \xrightarrow{\min} R = [y_{\min}, +\infty) \rightarrow y_{\min} = \frac{-\Delta}{2a} = \frac{-4}{2} = -2 \rightarrow R = [-2, +\infty)$

ب) $y = \sqrt{-x^2 + 4x - 2} \rightarrow y^2 = -x^2 + 4x - 2 \rightarrow x^2 - 4x + 2 + y^2 = 0 \rightarrow \Delta \geq 0 \rightarrow 16 - 4(2 + y^2) \geq 0 \rightarrow 4 - y^2 \geq 0 \rightarrow -2 \leq y \leq 2$

ج) $y = \sqrt{x^2 - 2x^2 + 4x + 1} \rightarrow R = \sqrt{IR} \Rightarrow R = [0, +\infty)$

$R = [0, 2]$

د) $y = x^2 - 4x^2 + 4x + 2 \rightarrow x^2 - 4x^2 + 4x + 2 > 0 \rightarrow R = (0, +\infty)$

الف) $y = \frac{x^2 + 1}{x - 2} \rightarrow R = IR - \left\{ \frac{1}{2} \right\} \rightarrow R = IR - \left\{ \frac{1}{2} \right\}$

ب) $y = \sqrt{\frac{x^2 + 1}{x - 2}} \rightarrow R = \sqrt{IR - \left\{ \frac{1}{2} \right\}} \rightarrow R = [0, +\infty) - \left\{ \frac{1}{2} \right\}$

ج) $y = x^2 + \frac{1}{x^2 + 2} \rightarrow y = x^2 + 2 + \frac{1}{x^2 + 2} - 2 \rightarrow R = \left[\frac{5}{2}, +\infty \right)$

$y = \frac{x^2 + 2}{x^2 - 1} \rightarrow (-12 + 4y)(2 + y) \geq 0$

$4y^2 + 2y - 12 \geq 0$

$y^2 + y - 3 \geq 0$

$\frac{-1}{+} \frac{3}{-} \rightarrow R = (-\infty, -3] \cup [2, +\infty)$

$x = \frac{0 \pm \sqrt{4 - (2 \pm (2 - y)) \cdot (2 + y)}}{2(2 - y)}$

$y \neq 2$

$R = (-\infty, -3] \cup [2, +\infty)$

$y = \frac{r \sin x - 1}{\sin x + r}$

$y \sin x + ry = r \sin x - 1$

$\sin x = \frac{1 + ry}{r - y}$

①, ② $\rightarrow R = \left[-\frac{r}{r}, \frac{1}{2} \right]$

$-1 \leq \frac{1 + ry}{r - y} \leq 1$

$\frac{ry + r}{r - y} \geq 0$

$\frac{-\frac{r}{r}}{-\frac{1}{r} + \frac{1}{r}} \rightarrow \left[-\frac{r}{r}, r \right]$

$\left[-\frac{r}{r}, r \right]$

$\frac{ry - 1}{r - y} \leq 0$

$\frac{\frac{1}{2} - r}{-\frac{1}{r} + \frac{1}{r}} \rightarrow (-\infty, \frac{1}{2}] \cup (r, +\infty)$

$(-\infty, \frac{1}{2}] \cup (r, +\infty)$

در صورتی که:

$\sin x = -1 \rightarrow y = \frac{r}{r}$

$\sin x = 1 \rightarrow y = \frac{1}{2}$

$R = \left[-\frac{r}{r}, \frac{1}{2} \right]$

$$f(x) = \frac{x^2 - x + 1}{x^2 - x + 1}$$

$$Df = \mathbb{R}$$

$$b^2 - 4ac \geq 0$$

$$y x^2 - y x + y = x^2 - x + 1$$

$$(1-y)^2 - (4x(y-1) \times (y - \frac{1}{x})) \geq 0$$

$$(y-1)x^2 + (1-y)x + y - \frac{1}{x} = 0$$

$$1+y^2 - 2y - (4y^2 - 4y + 1) \geq 0$$

$$1+y^2 - 2y - 4y^2 + 4y - 1 \geq 0$$

$$y \neq 1 \Delta \geq 0$$

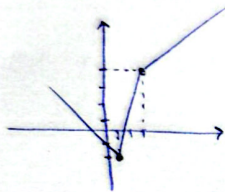
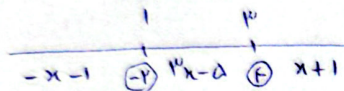
$$-3y^2 + 2y \geq 0$$

$$3y(-y+1) \geq 0$$

$$-1 + 1 = 0$$

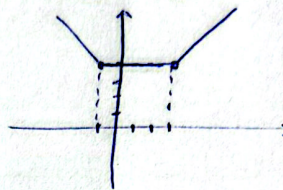
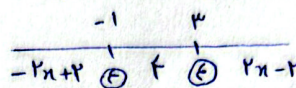
$$R_f = [0, 1)$$

$$a) y = |x-1| - |x-2|$$



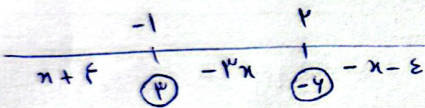
$$R = [-1, +\infty)$$

$$b) y = |x-1| + |x+1|$$



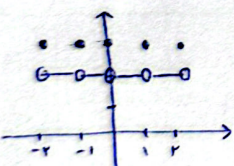
$$R = [2, +\infty)$$

$$|x-1| - |x+2| \leq 1$$



$$\left. \begin{array}{l} x+2 \leq 1 \\ x \leq -3 \end{array} \right\} \begin{array}{l} -3x \leq 1 \\ x \geq -\frac{1}{3} \end{array} \begin{array}{l} -x-2 \leq 1 \\ x \geq -3 \end{array} \rightarrow (-\infty, -3] \cup [-\frac{1}{3}, +\infty)$$

$$a) y = [x] + [-x] + 3$$



$$R = \{2, 3\}$$

$$b) y = 3x - [x]$$

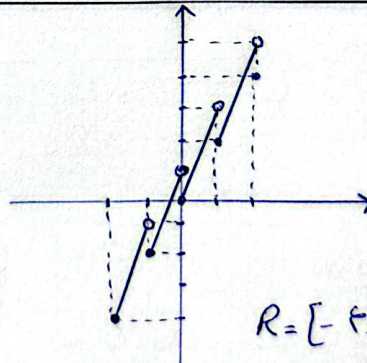
$$-1 \leq x < -1 \quad [x] = -1 \quad y = 3x + 1$$

$$-1 < x < 0 \quad [x] = 0 \quad y = 3x + 1$$

$$0 < x < 1 \quad [x] = 0 \quad y = 3x$$

$$1 \leq x < 2 \quad [x] = 1 \quad y = 3x - 1$$

$$x = 2 \quad [x] = 2 \quad y = 5$$



$$R = [-1, 5)$$

$$a) y = \sin^2 x - \sin x + 1$$

$$\begin{cases} \sin x = -1 \rightarrow y = 3 \\ \sin x = 1 \rightarrow y = 1 \\ \sin x = -\frac{b}{2a} = -\frac{1}{2} \rightarrow y = \frac{3}{2} \end{cases}$$

$$R = [\frac{3}{2}, 3]$$

$$b) y = \sin^2 x + \sin^2 x + 1$$

$$\begin{cases} \sin^2 x = 1 \rightarrow y = 3 \\ \sin^2 x = 0 \rightarrow y = 1 \\ \sin^2 x = -\frac{b}{2a} = -\frac{1}{2} \rightarrow \text{no solution} \end{cases}$$

$$R = [1, 3]$$