

$$y = \frac{x^2 + x + 2}{x - 1} \rightarrow yx - y = x^2 + x + 2 \rightarrow x^2 + (1 - y)x + 2 + y = 0 \rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$x = \frac{-1 + y \pm \sqrt{(1 - y)^2 - 4(2 + y)}}{2} \rightarrow (1 - y)^2 - (4 + 2y) \geq 0$$

$$y^2 - 4y - 2 \geq 0 \rightarrow (y + 1)(y - 5) \geq 0$$

$$\frac{-1}{+} \frac{5}{-} \rightarrow R = (-\infty, -1] \cup [5, +\infty)$$

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الف) $y = 2x^2 - 8x + 1 \xrightarrow{\min} R = [y_{\min}, +\infty) \rightarrow y_{\min} = \frac{-\Delta}{2a} = \frac{64 - 8}{8} = \frac{56}{8} = 7 \rightarrow R = [7, +\infty)$

ب) $y = \sqrt{-x^2 + 4x - 8} \rightarrow y^2 = -x^2 + 4x - 8 \rightarrow x^2 - 4x + 8 + y^2 = 0 \rightarrow \Delta \geq 0 \rightarrow 16 - (8 + 4y^2) \geq 0 \rightarrow 4y^2 \leq 8 \rightarrow -\sqrt{2} \leq y \leq \sqrt{2}$

ج) $y = \sqrt{x^2 - 2x^2 + 4x + 1} \rightarrow R = \sqrt{1R} \Rightarrow R = [0, +\infty)$

$R = [0, 2]$
برای $x=0 \rightarrow y=1$ و برای $x=2 \rightarrow y=1$

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الف) $y = \frac{x^2 + 1}{x - 2} \rightarrow R = \mathbb{R} - \left\{ \frac{5}{3} \right\} \rightarrow R = \mathbb{R} - \left\{ \frac{5}{3} \right\}$ ب) $y = \sqrt{\frac{x+1}{x-2}} \rightarrow R = \sqrt{\mathbb{R} - \left\{ \frac{5}{3} \right\}} \rightarrow R = [0, +\infty) - \left\{ \frac{5}{3} \right\}$

ج) $y = \frac{x^2 + 2}{\sqrt{x^2 + 3}} \rightarrow y = \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}} \rightarrow R = \left[\frac{2\sqrt{3}}{3}, +\infty \right)$

د) $y = x^2 + \frac{1}{x^2 + 2} \rightarrow y = x^2 + 2 + \frac{1}{x^2 + 2} - 2 \rightarrow R = \left[\frac{1}{2}, +\infty \right)$

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$y = \frac{x^2 + 2}{x^2 - 1} \rightarrow (-12 + 2y)(2 + y) \geq 0$

$yx^2 - y = x^2 + 2 \rightarrow (x - y)x^2 + 2y = 0$

$x = \frac{0 \pm \sqrt{4 - (2 \times (x - y)) \times (2 + y)}}{x - y}$

$y \neq 0$

$R = (-\infty, -2] \cup [2, +\infty)$

نکته:

$x = 0 \rightarrow y = -2$
 $x = +\infty \rightarrow y = 2$
خرج می‌دهد منفرد $(-\infty, -2] \cup [2, +\infty)$

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$y = \frac{\sin x - 1}{\sin x + 2}$

پیش فرض $\Rightarrow y \sin x + 2y = \sin x - 1$

$\sin x = \frac{1 + 2y}{1 - y}$

①, ② $\rightarrow R = \left[-\frac{3}{2}, \frac{1}{2} \right]$

$-1 \leq \frac{1 + 2y}{1 - y} \leq 1$

$\frac{1 + 2y}{1 - y} \geq -1$

$\frac{1 + 2y}{1 - y} \leq 1$

$\frac{1 + 2y}{1 - y} \geq -1$

$\left[-\frac{3}{2}, 1 \right)$ ①

$\frac{1 + 2y}{1 - y} \leq 1$

$\frac{1 + 2y}{1 - y} \leq 1$

$\frac{1 + 2y}{1 - y} \leq 1$

$(-\infty, \frac{1}{2}] \cup (2, +\infty)$ ②

نکته:

$\sin x = -1 \rightarrow y = \frac{2}{3}$
 $\sin x = 1 \rightarrow y = \frac{1}{2}$
خرج می‌دهد منفرد $\left[-\frac{3}{2}, \frac{1}{2} \right]$

$R = \left[-\frac{3}{2}, \frac{1}{2} \right]$

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$$f(x) = \frac{x^2 - x + 1}{x^2 - x + 1}$$

$$Df = \mathbb{R}$$

$$b^2 - 4ac \geq 0$$

$$y x^2 - y x + y = x^2 - x + 1$$

$$(1-y)^2 - (4x(y-1) \times (y-\frac{1}{x})) \geq 0$$

$$(y-1)x^2 + (1-y)x + y - \frac{1}{x} = 0$$

$$1+y^2 - 2y - (4y^2 - 4y + 1) \geq 0$$

$$1+y^2 - 2y - 4y^2 + 4y - 1 \geq 0$$

$$y \neq 1 \Delta \geq 0$$

$$-3y^2 + 2y \geq 0$$

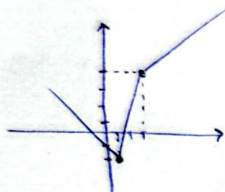
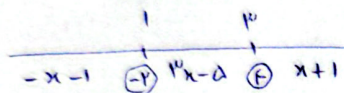
$$3y(-y+1) \geq 0$$

$$\frac{0}{-1} + \frac{1}{1} =$$

$$R_f = [0, 1)$$

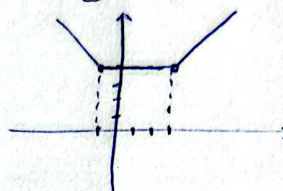
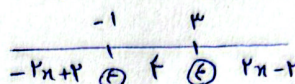
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$$a) y = |x-1| - |x-2|$$



$$R = [-1, +\infty)$$

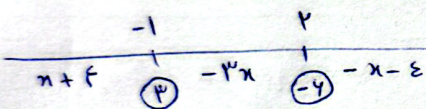
$$b) y = |x-1| + |x+1|$$



$$R = [2, +\infty)$$

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$$|x-1| - |x+2| \leq 1$$



$$x+2 \leq 1$$

$$-x \leq 1$$

$$-x-2 \leq 1$$

$$x \leq -3$$

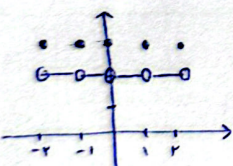
$$x \geq -1$$

$$x \geq -3$$

$$\} \rightarrow (-\infty, -3] \cup [-1, +\infty)$$

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$$a) y = [x] + [-x] + 3$$



$$R = \{3\}$$

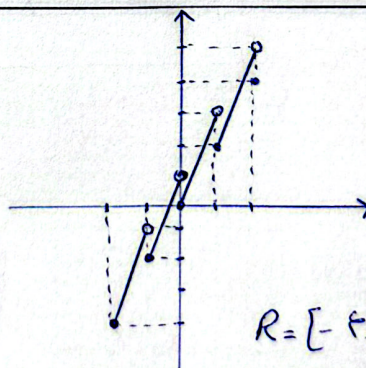
$$b) y = 3x - [x]$$

$$-1 \leq x < 0 \quad [x] = -1 \quad y = 3x + 1$$

$$0 \leq x < 1 \quad [x] = 0 \quad y = 3x$$

$$1 \leq x < 2 \quad [x] = 1 \quad y = 3x - 1$$

$$x = 2 \quad [x] = 2 \quad y = 5$$



$$R = [-1, 5)$$

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$$a) y = \sin^2 x - \sin x + 1$$

$$\begin{cases} \sin x = -1 \rightarrow y = 3 \\ \sin x = 1 \rightarrow y = 1 \\ \sin x = -\frac{b}{2a} = -\frac{1}{2} \rightarrow y = \frac{3}{2} \end{cases}$$

$$R = [\frac{3}{2}, 3]$$

$$b) y = \sin^2 x + \sin x + 1$$

$$\begin{cases} \sin^2 x = 1 \rightarrow y = 3 \\ \sin^2 x = 0 \rightarrow y = 1 \\ \sin^2 x = -\frac{b}{2a} = -\frac{1}{2} \rightarrow \text{no real solution} \end{cases}$$

$$R = [1, 3]$$

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