ژوین شوریان

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① $y=\frac{x^2+x+2}{x-1}$ $\quad yx-y=x^2+x+2 \to 0=x^2+x(1-y)+2+y$

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$x=\frac{y-1\pm\sqrt{y^2-2y+1-8-4y}}{2}=\frac{y-1\pm\sqrt{y^2-6y-7}}{2}\xrightarrow{\Delta\ge 0}(y-7)(y+1)\ge 0$

$\begin{array}{ccccc} & -1 & & 7 & \\ + & 0 & - & 0 & + \end{array}$ $\quad R_f=(-\infty,-1]\cup[7,+\infty)$

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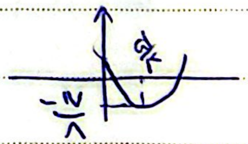
② الف) $y=2x^2-5x+1$ ext $\Big|$ $\frac{-b}{2a}=\frac{5}{4}$

$2(\frac{5}{4})^2-5(\frac{5}{4})+1=\frac{25}{8}-\frac{25}{4}+1=\frac{25-50+8}{8}=-\frac{17}{8}$



$R_f=[-\frac{17}{8},+\infty)$

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ب) $y=\sqrt{-x^2+6x-5}$ $\quad -x^2+6x-5\ge 0 \quad x^2-6x+5\le 0 \quad (x-5)(x-1)\le 0$

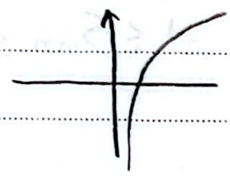
$\begin{array}{ccccc} & 1 & & 5 & \\ + & 0 & - & 0 & + \end{array}$ $\quad D_f=[1,5]$

R_f? $\quad$ ext $\Big|$ 3 $\quad -9+18-5=4$

$\sqrt{(-\infty,4]} \longrightarrow R_f=[0,2]$

ج) $y=\sqrt{x^7-3x^2+4x+1}$ $\xrightarrow{\text{در زیر رادیکال}}$ $\mathbb{R}$ $\xrightarrow{\sqrt{\mathbb{R}}}$ $R_f=[0,+\infty)$

د) $\log_{10}(x^3-6x^2+7x+4)$ $\xrightarrow{x^3-6x^2+7x+4>0}$



$R_f=\mathbb{R}$

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③ الف) $y=\frac{3x+1}{2x-6}=y$ $\quad x=-\frac{-6y-1}{2y-3}\Rightarrow x=\frac{6y+1}{2y-3}$ $\quad y\ne\frac{3}{2}$

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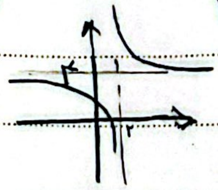
ب) $\sqrt{\frac{4x+1}{x-2}}$



$R_f=[0,+\infty)-\{4\}$ $\longrightarrow$ $[0,+\infty)-\{2\}$

ج) $y=\frac{x^2+4}{\sqrt{x^2+3}}$ $\quad \frac{x^2+3}{\sqrt{x^2+3}}+\frac{1}{\sqrt{x^2+3}}\Rightarrow\sqrt{x^2+3}+\frac{1}{\sqrt{x^2+3}}=y$

$\xrightarrow{\min}\sqrt{3}+\frac{1}{\sqrt{3}}$ $\quad \max=+\infty$ $\quad R_f=[\sqrt{3}+\frac{1}{\sqrt{3}},+\infty)$

Arman

۳) $y = x^2 + \frac{1}{x^2+4} \longrightarrow y = \underbrace{x^2+4+\frac{1}{x^2+4}}_{[4+\frac{1}{4}, +\infty)} - 4$

$\longrightarrow R_f = [4+\frac{1}{4}-4, +\infty - 4] \Rightarrow R_f = [\frac{1}{4}, +\infty)$

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(۴) $y = \frac{3x^2+4}{x^2-1}$ $\quad yx^2 - y = 3x^2+4 \longrightarrow (3-y)x^2+4+y=0$

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$x = \frac{-0 \pm \sqrt{-4(4+y)(3-y)}}{3-y} = \begin{cases} \xrightarrow{y=3} V=0 \quad \text{غ ق ق} \\ \xrightarrow{\Delta \ge 0} \begin{array}{ccccc} & -4 & & 3 & \\ \hline + & 0 & - & \text{غ} & + \end{array} \end{cases} \rightarrow R_f = \mathbb{R} - (-4, 3]$

$y = \frac{3x^2+4}{x^2-1} \begin{cases} \xrightarrow{x^2=0} y = -4 \\ \xrightarrow{x^2=+\infty} y \simeq 3 \end{cases}$ $\xrightarrow{\text{طرح منحنی نمودار}} R_f = (-\infty, -4] \cup (3, +\infty)$

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(۵) $y = \frac{2\sin x - 1}{\sin x + 2} = y$ $\quad \sin x = -\frac{+3y+1}{y-2} = \frac{3y+1}{2-y}$

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$-1 \le \sin x \le 1 \to -1 \le \frac{3y+1}{2-y} \le 1 \longrightarrow \frac{3y+1+y-2}{2-y} \le 0 \to \frac{4y-1}{2-y} \le 0$

$\begin{array}{ccccc} & \frac{1}{4} & & 2 & \\ \hline - & 0 & + & \text{غ} & - \end{array}$

$\to 0 \le \frac{3y+1+2-y}{2-y} \to 0 \le \frac{2y+3}{2-y}$

$\begin{array}{ccccc} & -\frac{3}{2} & & 2 & \\ \hline - & 0 & + & \text{غ} & - \end{array}$

① ∩ ② $\longrightarrow R_f \ [-\frac{3}{2} \quad \frac{1}{4}]$

$\left.\begin{array}{l} \xrightarrow{\sin x = 1} y = \frac{2-1}{4} = \frac{1}{4} \\ \xrightarrow{\sin x = -1} y = \frac{-3}{2} = \frac{-3}{2} \end{array}\right\} \Rightarrow R_f = [-\frac{3}{2} \quad \frac{1}{4}]$

(۶) ۲

$$f(x)=\frac{x^2-x+\frac{1}{4}}{x^2-x+1}$$

$x^2-x+1\neq 0 \xrightarrow{\Delta<0} 1-4<0$ ✓ $D_f=\mathbb{R}$ ✓

$$y=\frac{x^2-x+\frac{1}{4}}{x^2-x+1} \longrightarrow yx^2-yx+y=x^2-x+\frac{1}{4}$$

$(y-1)x^2+(1-y)x+y-\frac{1}{4}=0$

$\xrightarrow{\Delta\geq 0} (1-y)^2-4(y-1)(y-\frac{1}{4})\geq 0$

$y^2-2y+1-4(y^2-\frac{5}{4}y+\frac{1}{4})\geq 0$

$y^2-2y+1-4y^2+5y-1\geq 0$

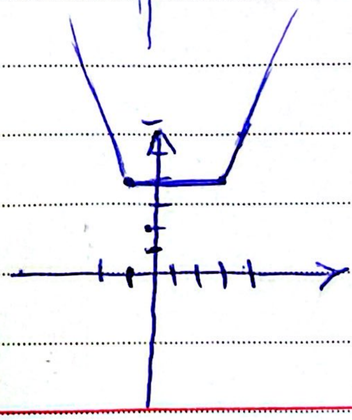
$-3y^2+3y\geq 0 \longrightarrow 3y(1-y)\geq 0$ $\quad \begin{array}{ccccc} & 0 & & 1 & \\ - & & + & & - \end{array}$

$\xrightarrow{y\neq 1} R_f=[0,1)$ ✓

(۷) ۲

الف) $y=|2x-2|-|x-3|$ $\quad$ 1 $\quad$ 3

$R_f=[-2,+\infty)$



ب) $y=|x-3|+|x+1|$

$R_f=[4,+\infty)$

(۸) $|x-2|-|2x+2|\leq 1$

$-1 \qquad 2$

$-x+2+2x+2\leq 1 \;\big|\; 2-x-2x-2 \;\big|\; -x-4\leq 1$

$x+4\leq 1 \qquad -3x\leq 1 \qquad -5\leq x$ $\qquad D_x:(-\infty,-1]\cup[-\frac{1}{3},2)$

$x\leq -3 \qquad x\geq -\frac{1}{3}$

$x\in(-\infty,-3]\cup[-\frac{1}{3},+\infty)$

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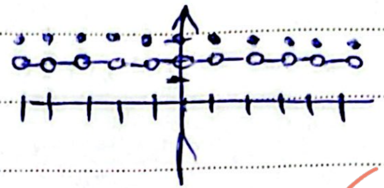
$y = [x] + [-x] + 3$

$x = \mathbb{Z} \rightarrow y = 3$

$x \neq \mathbb{Z} \rightarrow y = 2$



$R_f = \{2, 3\}$

ب) $y = 3x - [x]$

0/5

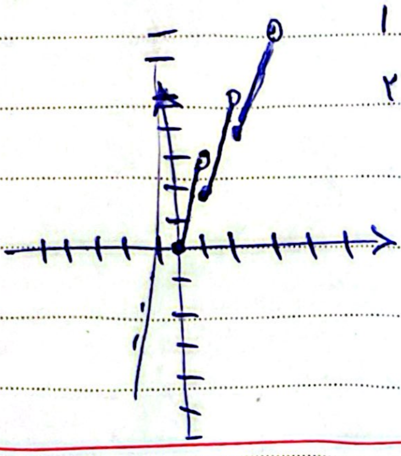
$0 \le x < 1 \quad y = 3x$

$1 \le x < 2 \quad y = 3x - 1$

$2 \le x < 3 \quad y = 3x - 2$

دامنه $[-2, 2]$

$[-6, 0)$



طبق الگو $\rightarrow R_f = \mathbb{R}$

⑩ الف) $\sin^2 x - \sin x + 1 = y$

1/0

$\xrightarrow{\sin x = 1} y = 1$

$\xrightarrow{\sin x = -1} y = 3$

$\xrightarrow{-\frac{b}{2a} = \frac{1}{2}} \frac{1}{4} - \frac{1}{2} + 1 = \frac{5}{4}$

$R_f = [1 \quad 3]$

1

ب) $\sin^4 x + \sin^2 x + 1 = y$

0/5

$\xrightarrow{\sin x = 1} y = 3$

$\xrightarrow{\sin x = -1} y = 3$

$\xrightarrow{-\frac{b}{2a} = -\frac{1}{2}} \frac{1}{16} + \frac{1}{4} + 1 = \frac{21}{16}$

$R_f = [\frac{21}{16} \quad 3]$

$[1, 3]$

$\sin^2 x \neq -\frac{1}{2}$