روژین شسودیان

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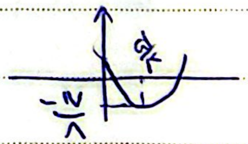
① $y=\frac{x^2+x+2}{x-1}$ $yx-y=x^2+x+2 \longrightarrow 0=x^2+x(1-y)+2+y$

$x=\frac{y-1\pm\sqrt{y^2-2y+1-8-4y}}{2}=\frac{y-1\pm\sqrt{y^2-6y-7}}{2} \xrightarrow{\Delta\geq 0} (y-7)(y+1)\geq 0$

$\begin{array}{ccccc} & -1 & & 7 & \\ + & 0 & - & 0 & + \end{array}$ $R_f=(-\infty,-1]\cup[7,+\infty)$

② الف) $y=2x^2-5x+1$ ext | $\frac{-b}{2a}=\frac{5}{4}$

$2(\frac{5}{4})^2-5(\frac{5}{4})+1=\frac{25}{8}-\frac{25}{4}+1=\frac{25-50+8}{8}=-\frac{17}{8}$



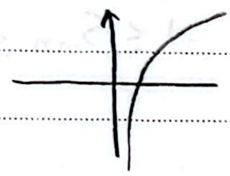

$R_f=[-\frac{17}{8},+\infty)$

ب) $y=\sqrt{-x^2+6x-5}$ $-x^2+6x-5\geq 0$ $x^2-6x+5\leq 0$ $(x-5)(x-1)\leq 0$

$\begin{array}{ccccc} & 1 & & 5 & \\ + & 0 & - & 0 & + \end{array}$ $D_f=[1\ \ 5]$

ج) $y=\sqrt{x^7-3x^2+4x+1}$ $\xrightarrow{\text{برد زیر رادیکال}} \mathbb{R} \xrightarrow{\sqrt{\mathbb{R}}} R_f=[0,+\infty)$

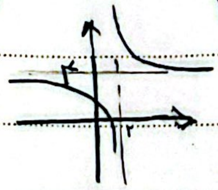
د) $\log_{10}^{x^3-6x^2+7x+4}$ $\xrightarrow{x^3-6x^2+7x+4>0}$



$R_f=\mathbb{R}$

③ الف) $y=\frac{3x+1}{2x-6}=y$ $x=-\frac{-6y-1}{2y-3} \Rightarrow x=\frac{6y+1}{2y-3}$ $y\neq\frac{3}{2}$

ب) $\sqrt{\frac{4x+1}{x-2}}$ $\frac{1}{-2}$




$R_f=[0,+\infty)-\{4\}$

ج) $y=\frac{x^2+4}{\sqrt{x^2+3}}$ $\frac{x^2+3}{\sqrt{x^2+3}}+\frac{1}{\sqrt{x^2+3}} \Rightarrow \sqrt{x^2+3}+\frac{1}{\sqrt{x^2+3}}=y$

$\xrightarrow{min} \sqrt{3}+\frac{1}{\sqrt{3}}$ $max=+\infty$ $R_f=[\sqrt{3}+\frac{1}{\sqrt{3}},+\infty)$

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د) $y = x^2 + \frac{1}{x^2+4} \longrightarrow y = \underbrace{x^2+4 + \frac{1}{x^2+4}}_{[4+\frac{1}{4},\ +\infty)} - 4$

$\longrightarrow R_f = [4+\frac{1}{4} - 4,\ +\infty - 4] \Rightarrow R_f = [\frac{1}{4},\ +\infty)$

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۴) $y = \frac{3x^2+4}{x^2-1} \qquad yx^2 - y = 3x^2 + 4 \longrightarrow (3-y)x^2 + 4 + y = 0$

$x = \frac{-0 \pm \sqrt{-4(4+y)(3-y)}}{3-y} = \begin{cases} \xrightarrow{y=3} V=0 \quad \text{غ ق ق} \\ \xrightarrow{\Delta \geq 0} \begin{array}{ccccc} & -4 & & 3 & \\ + & 0 & - & \cancel{0} & + \end{array} \end{cases} \longrightarrow R_f = R - (-4, 3]$

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$y = \frac{3x^2+4}{x^2-1} \begin{cases} \xrightarrow{x^2=0} y = -4 \\ \xrightarrow{x^2=+\infty} y \simeq 3 \end{cases}$ $\xrightarrow{\text{از رسم نمودار}} R_f = (-\infty, -4] \cup (3, +\infty)$

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۵) $y = \frac{2\sin x - 1}{\sin x + 2} = y \qquad \sin x = -\frac{+3y+1}{y-2} = \frac{3y+1}{2-y}$

$-1 \leq \sin x \leq 1 \rightarrow -1 \leq \frac{3y+1}{2-y} \leq 1 \longrightarrow \frac{3y+1+y-2}{2-y} \leq 0 \longrightarrow \frac{4y-1}{2-y} \leq 0$

$\begin{array}{ccccc} & \frac{1}{4} & & 2 & \\ - & 0 & + & \cancel{0} & - \end{array}$

$\longrightarrow 0 \leq \frac{3y+1+2-y}{2-y} \longrightarrow 0 \leq \frac{2y+3}{2-y}$

$\begin{array}{ccccc} & -\frac{3}{2} & & 2 & \\ - & 0 & + & \cancel{0} & - \end{array}$

① ∩ ② $\longrightarrow R_f\ [-\frac{3}{2}\ \ \frac{1}{4}]$

$\left.\begin{array}{l} \xrightarrow{\sin x = 1} y = \frac{2-1}{4} = \frac{1}{4} \\ \xrightarrow{\sin x = -1} y = \frac{-3}{2} = \frac{-3}{2} \end{array}\right\} \Rightarrow R_f = [-\frac{3}{2}\ \ \frac{1}{4}]$

(6) $f(x) = \dfrac{x^2 - x + \frac{1}{4}}{x^2 - x + 1}$

$x^2 - x + 1 \neq 0 \xrightarrow{\Delta < 0} 1 - 4 < 0$ ✓ $D_f = \mathbb{R}$

$y = \dfrac{x^2 - x + \frac{1}{4}}{x^2 - x + 1} \longrightarrow yx^2 - yx + y = x^2 - x + \frac{1}{4}$

$(y-1)x^2 + (1-y)x + y - \frac{1}{4} = 0$

$\xrightarrow{\Delta \geq 0} (1-y)^2 - 4(y-1)(y - \frac{1}{4}) \geq 0$

$y^2 - 2y + 1 - 4(y^2 - \frac{5}{4}y + \frac{1}{4}) \geq 0$

$y^2 - 2y + 1 - 4y^2 + 5y - 1 \geq 0$

$-3y^2 + 3y \geq 0 \longrightarrow 3y(1-y) \geq 0$

| | 0 | | 1 | |
|---|---|---|---|---|
| $-$ | | $+$ | | $-$ |

$\xrightarrow{y \neq 1} \mathbb{R}_f = [0, 1)$

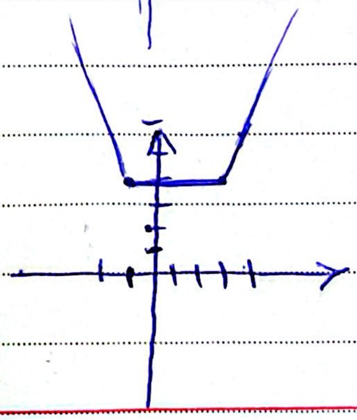
(7) الف) $y = |2x - 2| - |x - 3|$

$\quad\quad\quad\quad\quad\quad 1 \quad\quad\quad 3$

$R_f = [-2, +\infty)$

ب) $y = |x - 3| + |x + 1|$

$R_f = [4, +\infty)$



(8) $|x - 2| - |2x + 2| \leq 1$

$\quad -1 \quad\quad 2$

| $-x + 2 + 2x + 2$ | $2 - x - 2x - 2$ | $-x - 4 \leq 1$ |
|---|---|---|
| $x + 4 \leq 1$ | $-3x \leq 1$ | $-5 \leq x$ |
| $x \leq 3$ | $x \geq \frac{-1}{3}$ | |

$D_x : (-\infty, -1] \cup [-\frac{1}{3}, 2)$

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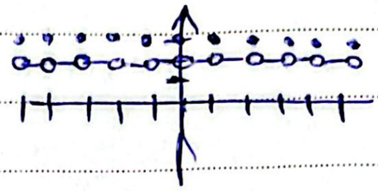
$$y = [x] + [-x] + 3 \quad \begin{cases} \xrightarrow{x = \mathbb{Z}} y = 3 \\ \xrightarrow{x \neq \mathbb{Z}} y = 2 \end{cases}$$



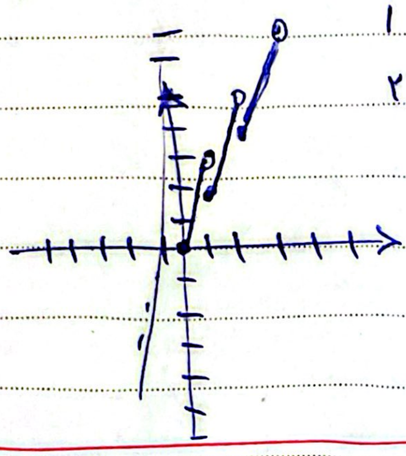
$R_f = \{2, 3\}$

ب) $y = 3x - [x]$

$0 \le x < 1 \quad y = 3x$

$1 \le x < 2 \quad y = 3x - 1$

$2 \le x < 3 \quad y = 3x - 2$



$\xrightarrow{\text{صعودی اکید}} R_f = \mathbb{R}$

۱۰ الف) $\sin^2 x - \sin x + 1 = y$

$\xrightarrow{\sin x = 1} y = 1$

$\xrightarrow{\sin x = -1} y = 3$

$\xrightarrow{-\frac{b}{2a} = \frac{1}{2}} \frac{1}{4} - \frac{1}{2} + 1 = \frac{5}{4}$

$R_f = [1 \quad 3]$

ب) $\sin^4 x + \sin^2 x + 1 = y$

$\xrightarrow{\sin x = 1} y = 3$

$\xrightarrow{\sin x = -1} y = 3$

$\xrightarrow{-\frac{b}{2a} = -\frac{1}{4}} \frac{1}{16} + \frac{1}{4} + 1 = \frac{21}{16}$

$R_f = [\frac{21}{16} \quad 3]$

Arman