

$$y = \frac{x^2 + x + 2}{x - 1} \rightarrow yx - y = x^2 + x + 2$$

$$\Rightarrow x^2 + (1 - y)x + 2 + y = 0 \rightarrow x = \frac{-(1 - y) \pm \sqrt{\Delta}}{2 \times 1} \xrightarrow{\Delta \geq 0} \Delta \geq 0 \Rightarrow (1 - y)^2 - 4(2 + y) \geq 0$$

$$y^2 - 2y + 1 - 8 - 4y \geq 0 \rightarrow y^2 - 6y - 7 \geq 0 \rightarrow (y - 7)(y + 1) \geq 0 \rightarrow y \leq -1 \text{ یا } y \geq 7$$

$$R_f = (-\infty, -1] \cup [7, +\infty)$$

الف) $y = 2x^2 - 5x + 1$ $\rightarrow$ $\Delta = 25 - 8 = 17$ $\rightarrow$ $x = \frac{5 \pm \sqrt{17}}{4}$ $\rightarrow$ $y = \frac{17}{8}$ $\rightarrow$ $R = [\frac{17}{8}, +\infty)$

ب) $y = \sqrt{-x^2 + 4x - 5}$ $\rightarrow$ $-x^2 + 4x - 5 \geq 0$ $\rightarrow$ $x^2 - 4x + 5 \leq 0$ $\rightarrow$ $\Delta = 16 - 20 = -4 < 0$ $\rightarrow$ $R = \emptyset$

ج) $y = \frac{x^2 + 1}{x^2 - 2}$ $\rightarrow$ $y(x^2 - 2) = x^2 + 1$ $\rightarrow$ $(y - 1)x^2 - 2y = 0$ $\rightarrow$ $x^2 = \frac{2y}{y - 1}$ $\rightarrow$ $\frac{2y}{y - 1} \geq 0$ $\rightarrow$ $R = (-\infty, 0) \cup [2, +\infty)$

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$f(n) = \frac{n^2 - n + \frac{1}{2}}{n^2 - n + 1}$ $\neq 2$ $\rightarrow n^2 - n + 1 \neq 0$ $\left[\begin{matrix} a > 0 \\ \Delta < 0 \\ (1-a)^2 < 0 \end{matrix} \right] \Rightarrow D_f = \mathbb{R}$

$y = \frac{n^2 - n + \frac{1}{2}}{n^2 - n + 1} \rightarrow yn^2 - yn + y = n^2 - n + \frac{1}{2}$

$\rightarrow (1-y)n^2 + (y-1)n + \frac{1}{2} - y = 0 \rightarrow n = \frac{-(y-1) \pm \sqrt{\Delta}}{2(1-y)} \rightarrow \Delta \geq 0 \rightarrow (y-1)^2 - 2(1-y)(1-y) \geq 0$

$\Delta = b^2 - 4ac$ $\rightarrow (y-1)^2 - 2(1-y)(1-y) \geq 0$ $\rightarrow (y-1)^2 - 2(1-y)^2 \geq 0$

$\rightarrow 1-y \neq 0 \rightarrow y \neq 1$

الف) $y = |n-2| - |n-3|$

$\begin{matrix} 2 & 3 \\ \downarrow & \downarrow \\ -n+2 & -n+3 \\ -n+2 & -n+3 \\ -n+2 & -n+3 \end{matrix}$

$\mathbb{R} \Rightarrow [-2, +\infty)$

ب) $y = |n-3| + |n+1|$

$\begin{matrix} 3 & -1 \\ \downarrow & \downarrow \\ -n+3 & -n-1 \\ -n+3 & -n-1 \\ -n+3 & -n-1 \end{matrix}$

$\mathbb{R} \Rightarrow [4, +\infty)$

$y = |n-2| - |n+2| \leq 1$

$\begin{matrix} -1 & 2 \\ \downarrow & \downarrow \\ -n-1 & -n+2 \\ -n-1 & -n+2 \\ -n-1 & -n+2 \end{matrix}$

$n=2 \rightarrow y=-4$ $n=-1 \rightarrow y=3$ $n=0 \rightarrow y=0$ $n=-2 \rightarrow y=2$

$\Rightarrow x \in \mathbb{R} - (-\frac{1}{2}, \frac{1}{2})$

الف) $y = [n] + [\frac{n}{2}]$ $\mathbb{R} = \{2, 3\}$

$y = 2 + [n] + [-n]$ $n \in \mathbb{Z} \rightarrow 0$ $n \notin \mathbb{Z} \rightarrow -1$

$y = [n] + [-n]$ $n \in \mathbb{Z} \rightarrow y=0$ $n \notin \mathbb{Z} \rightarrow y=-1$

ب) $y = 3 + [n] + [-n]$

$-2 < n < -1 \rightarrow y = 3n+2 \rightarrow n=-1 \rightarrow y=-1$ $-1 < n < 0 \rightarrow y = 3n+1 \rightarrow n=0 \rightarrow y=1$ $0 \leq n < 1 \rightarrow y = 3n \rightarrow n=0 \rightarrow y=0$ $1 < n < 2 \rightarrow y = 3n-1 \rightarrow n=1 \rightarrow y=2$ $n=2 \rightarrow y=5$

$\mathbb{R} \Rightarrow [-1, 5)$

الف) $y = \sin^2 n - \sin n + 1$

$\sin n \rightarrow \max = 1 \rightarrow 1-1+1=1$ $\min = -1 \rightarrow 1+1+1=3$

$\frac{-b}{2a} \rightarrow \frac{1}{2} \rightarrow \frac{1}{2} - \frac{1}{2} + 1 = \frac{1}{2}$

$\mathbb{R} = [\frac{1}{2}, 3]$

ب) $y = \sin^2 n + \sin n + 1$

$\sin n \rightarrow t$ $t \rightarrow \max \rightarrow 1 \rightarrow 1+1+1=3$ $\min \rightarrow 0 \rightarrow 0+0+1=1$

$\frac{-b}{2a} \Rightarrow \frac{-1}{2} \Rightarrow \frac{3}{4}$

$\mathbb{R}_f = [1, 3]$