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$$y^2 - y = m^2 + m + 2 \Rightarrow m^2 + (1-y)m + y + 2 = 0 \Rightarrow m = \frac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow m = \frac{y-1 \pm \sqrt{y^2 + 1 - 2y - 4(1)(y+2)}}{2}$$

$$\Rightarrow \Delta \geq 0 \Rightarrow y^2 + 1 - 2y - 4y - 8 \geq 0 \Rightarrow y^2 - 4y - 7 \geq 0$$

$$\Rightarrow (y-7)(y+1) \geq 0 \Rightarrow R_f = (-\infty, -1] \cup [7, +\infty)$$

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الف) $y = 2m^2 - 5m + 1$
 تابع دایره ای $\Rightarrow \min$ تابع دایره ای $\Rightarrow \min = \frac{-b}{2a} = \frac{5}{4}$
 $\Rightarrow y = 2(\frac{5}{4})^2 - 5(\frac{5}{4}) + 1 = -\frac{13}{8}$
 $\Rightarrow R_f = [-\frac{13}{8}, +\infty)$

ب) $y = \frac{m^2 + 1}{m - 1} \Rightarrow \max$ تابع دایره ای $\Rightarrow \max = \frac{-b}{2a} = \frac{4}{2} = 2$
 $\Rightarrow y = \frac{2^2 + 1}{2 - 1} = 5$
 $\Rightarrow R_f = (-\infty, 5]$

ج) $R_f = \mathbb{R} \Rightarrow \sqrt{R} \Rightarrow R_f = [0, +\infty)$

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الف) $y = \frac{m^2 + 1}{m - 1} \Rightarrow R_f = \mathbb{R} \Rightarrow \sqrt{R} \Rightarrow R_f = [0, +\infty)$

ب) $y = \sqrt{\frac{m^2 + 1}{m - 1}} \Rightarrow R_f = \sqrt{R} \Rightarrow R_f = [0, +\infty) - \{1\}$

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الف) $y = \frac{m^2 + 1}{\sqrt{m^2 + 3}} \Rightarrow y = \frac{m^2 + 1}{\sqrt{m^2 + 3}} \Rightarrow \frac{m^2 + 1}{\sqrt{m^2 + 3}} = \sqrt{m^2 + 3} + \frac{1}{\sqrt{m^2 + 3}} \Rightarrow R_f = [2, +\infty)$

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الف) $y = \frac{m^2 + 1}{m - 1} \Rightarrow y^2 - y = m^2 + 1 \Rightarrow y^2 - y - m^2 - 1 = 0 \Rightarrow m^2 = y^2 - y - 1$
 $\Rightarrow m^2 \geq 0 \Rightarrow y^2 - y - 1 \geq 0 \Rightarrow R_f = (-\infty, -1] \cup [2, +\infty)$

ب) $y = \frac{m^2 + 1}{m - 1} \Rightarrow \min$ تابع دایره ای $\Rightarrow \min = \frac{-b}{2a} = \frac{4}{2} = 2$
 $\Rightarrow y = \frac{2^2 + 1}{2 - 1} = 5$
 $\Rightarrow R_f = (-\infty, 5]$

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الف) $y = \frac{1 - \sin x}{\sin x + 1} \Rightarrow y \sin x + 1 = 1 - \sin x \Rightarrow y \sin x - 1 \sin x = -1 - 1 \Rightarrow \sin x (y - 1) = -2$
 $\Rightarrow \sin x = \frac{-2}{y - 1} \Rightarrow -1 \leq \frac{-2}{y - 1} \leq 1 \Rightarrow \frac{1}{2} \leq y \leq \frac{3}{2} \Rightarrow R_f = [\frac{1}{2}, \frac{3}{2}]$

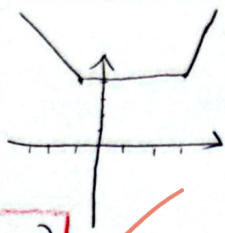
ب) $\min \sin x = -1 \Rightarrow \frac{1 - (-1)}{-1 + 1} = \frac{2}{0} \Rightarrow \frac{1}{2}$
 $\max \sin x = 1 \Rightarrow \frac{1 - (1)}{1 + 1} = \frac{0}{2} = 0 \Rightarrow \frac{3}{2}$
 $\Rightarrow R_f = [\frac{1}{2}, \frac{3}{2}]$

الف) $y = |x-1| - |x-2| \Rightarrow$ تابع با ضابطه $\Rightarrow \frac{x-1-x+2}{x+1} > 1 \Rightarrow \frac{x-2-x+1}{x-0} < 1$

ب) $y = |x-1| + |x+1|$

$\Rightarrow$ تابع V شکل $\Rightarrow$ رسم شده

رشته های درین $\Rightarrow x=1$



$R_f = [1, +\infty)$

$y = \frac{x-2-x+1}{x-0} < 1$



$R_f = [-1, +\infty)$

$y = \frac{x^2 - x + \frac{1}{x}}{x^2 - x + 1} \Rightarrow D_f = R \Rightarrow$ زیرا $\Delta < 0 \Rightarrow 1 - 1 + 1 < 0$

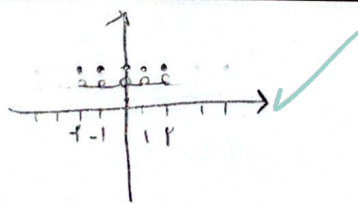
$y = \frac{x^2 - x + \frac{1}{x}}{x^2 - x + 1} \Rightarrow \frac{(x - \frac{1}{x})^2}{(x - \frac{1}{x})^2 + \frac{4}{x^2}} \Rightarrow (x - \frac{1}{x})^2 = t \Rightarrow t \geq 0 \Rightarrow y = \frac{t}{t + \frac{4}{x^2}} \Rightarrow \min t = 0 \Rightarrow 0$
 $\max t = \infty \Rightarrow 1$
 $\Rightarrow R_f = [0, 1]$

الف) $|x-1| - |x+1| \leq 1 \Rightarrow$ تابع با ضابطه $\Rightarrow$

$x-1-x-1 \leq 1 \Rightarrow -2 \leq 1$ $-x \leq -1 \Rightarrow x \geq 1$	$x-1-x+1 \leq 1 \Rightarrow 0 \leq 1$ $-x \leq -\frac{1}{x} \Rightarrow x \leq -\frac{1}{x}$ $x \leq -1$	$x-1-x-1 \geq 1 \Rightarrow -2 \geq 1$ $-x \geq 1 \Rightarrow x \leq -1$
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 $\Rightarrow x \in (-\infty, -1] \cup [1, +\infty)$

الف) $y = [x] + [-x] + x = \begin{cases} x & x \in \mathbb{Z} \\ 0 & x \notin \mathbb{Z} \end{cases}$



بانهایی که و فعاله ها بینویس

ب) $y = 2x - [x] \Rightarrow x \in (-1, 1) \Rightarrow y \in (-1, 1)$
 $x \in (-1, 0) \Rightarrow y \in (-1, 0)$
 $x \in (0, 1) \Rightarrow y \in (0, 1)$
 $R_f = R$



$y \in (1, 2) \Rightarrow x \in (1, 2)$
 $y \in (0, 1) \Rightarrow x \in (0, 1)$
 $y \in (-1, 0) \Rightarrow x \in (-1, 0)$
 $y \in (-2, -1) \Rightarrow x \in (-2, -1)$

الف) $y = \sin^2 x - \sin x + 1$
 $\max y = 1 - 1 + 1 = 1$
 $\min y = 1 + 1 + 1 = 3$
 $\frac{-b}{2a} = \frac{1}{2} - \frac{1}{2} + 1 = \frac{1}{2}$
 $R_f = [\frac{1}{2}, 3]$

ب) $y = \sin^2 x + \sin^2 x + 1$
 $\max y = (1)^2 + 1 + 1 = 3$
 $\min y = (0)^2 + 0 + 1 = 1$
 $R_f = [1, 3]$