

$$yx - y = x^2 + x + 2 \Rightarrow x^2 + (1-y)x + 2+y$$

$$\Delta = (1-y)^2 - 4(2+y) = y^2 - 2y + 1 - 8 - 4y = y^2 - 6y - 7$$

$$x = \frac{-(1-y) \pm \sqrt{y^2 - 6y - 7}}{2} \Rightarrow \frac{(y+1)(y-7)}{2} \quad R_f = (-\infty, -1] \cup (7, +\infty)$$

الف) $y = \frac{-\Delta}{f_a} = -\frac{(y^2 - 6y - 7)}{2} = -\frac{1}{2} \quad R_f = \left[-\frac{1}{2}, +\infty\right)$

ب) $y = \sqrt{(-\infty, 4]} \Rightarrow R_f = [0, 2]$

ج) $y = \sqrt{R} \rightarrow (-\infty, +\infty) = R_f$

د) $\log(x^2 - 6x + 7) \rightarrow R = R \Rightarrow R_f = (-\infty, +\infty)$

الف) $R_f = R - \left\{\frac{3}{2}\right\}$

ب) $y = \sqrt{R - \left\{\frac{3}{2}\right\}} \Rightarrow R_f = (-\infty, +\infty) - \left\{\frac{3}{2}\right\}$

ج) $\frac{x^2 + 3 + 1}{\sqrt{x^2 + 3}} = \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}} \quad \text{د) } x^2 + 3 + \frac{1}{x^2 + 3} - 4 = y \quad \frac{1}{4} - \frac{1}{4} = \frac{1}{4}$
 $\sqrt{3} + \frac{1}{\sqrt{3}} - \frac{4}{\sqrt{3}} \quad R_f = \left[\frac{4}{\sqrt{3}}, +\infty\right) \quad R_f = \left[\frac{1}{4}, +\infty\right)$

الف) $y = \frac{3x^2 + 4}{x^2 - 1} \Rightarrow yx^2 - y = 3x^2 + 4 \Rightarrow (y-3)x^2 + 4 + y = 0$

$\Delta = 0 - 4(y-3)(4+y)$

$x = 0 \pm \sqrt{4(y^2 + y - 12)}$

ب) $x^2 = 0 \Rightarrow y = -\frac{4}{3} \quad x^2 = +\infty \Rightarrow y = 3 \Rightarrow (-\infty, -\frac{4}{3}] \cup (3, +\infty) \quad R_f = (-\infty, -\frac{4}{3}] \cup (3, +\infty)$

الف) $\sin x = 1 \Rightarrow y = \frac{1}{4}$
 ب) $\sin x = -1 \Rightarrow y = -\frac{3}{4}$
 نتیجه: $\left[-\frac{3}{4}, \frac{1}{4}\right] = R_f$

الف) $y \sin x + 3y = 2 \sin x - 1 \Rightarrow \sin x = \frac{1+3y}{1-y}$

$-1 \leq \frac{1+3y}{1-y} \leq 1 \Rightarrow \frac{1+3y}{1-y} \leq 1 \quad \frac{1+3y}{1-y} \geq -1$
 $\frac{3}{4} < y < 1 \quad y < \frac{1}{4} \Rightarrow R_f = \left[-\frac{3}{4}, \frac{1}{4}\right]$

$$f(x) = \frac{x^2 - x + \frac{1}{x}}{x^2 - x + 1}$$

$$x^2 - x + 1 \neq 0 \quad D_f = \mathbb{R} \quad R_f = (-\infty, 1] \cup \left(\frac{1}{3}, \infty\right)$$

$$-3y^2 + 4y \geq 0 \Rightarrow \frac{0}{-3} \leq y \leq \frac{4}{-3}$$

$$y x^2 - y x + y = x^2 - x + \frac{1}{x} \Rightarrow (1-y)x^2 + (y-1)x + \frac{1}{x} - y = 0$$

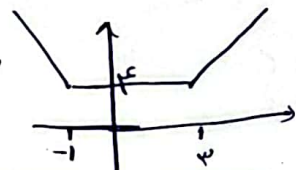
$$\Delta = (y-1)^2 - 4(1-y)\left(\frac{1}{x} - y\right) = -3y^2 + 4y \quad x = \frac{-(y-1) \pm \sqrt{-3y^2 + 4y}}{2(1-y)}$$

الف) $\begin{cases} x+1 & x \geq 3 \\ 3x-4 & 1 < x < 3 \\ -x-1 & x \leq 1 \end{cases}$

$$R_f = [-2, +\infty)$$



ب) $\begin{cases} 2x-2 & x \geq 3 \\ 2 & 1 < x < 3 \\ -2x+2 & x \leq 1 \end{cases}$

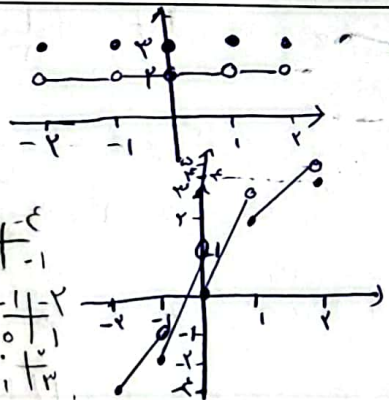


$$R_f = [2, +\infty)$$

	$x+1$	$3x-4$	$-x-1$
$x+1 \leq 1$	$x+1 \leq 1$		
$x+1 > 1$		$3x-4 \leq 1$	
$x+1 > 1$		$3x-4 > 1$	
$x+1 > 1$			$-x-1 \leq 1$
$x+1 > 1$			$-x-1 > 1$

$$(-\infty, -3] \cup \left[\frac{1}{3}, +\infty\right)$$

الف) $\lfloor x \rfloor + \lfloor -x \rfloor + 2$



$$R_f = [2, 3]$$

ب) $2x - \lfloor x \rfloor$

$$R_f = (-1, \infty)$$

$-1 < x < 0 \Rightarrow y = 2x + 2$
 $0 < x < 1 \Rightarrow y = 2x + 1$
 $1 < x < 2 \Rightarrow y = 2x$
 $2 < x < 3 \Rightarrow y = 2x - 1$
 $x = 2 \Rightarrow y = 2$

الف) $\sin \begin{cases} 1 & x \in [0, 1] \\ -1 & x \in [1, 2] \\ 0 & x \in [2, 3] \end{cases}$

$$R_f = \left[\frac{1}{2}, 3\right]$$

ب) $\sin^2 \begin{cases} 1 & x \in [0, 1] \\ 0 & x \in [1, 2] \\ 1 & x \in [2, 3] \end{cases}$

$$R_f = [1, 3]$$