

$$y = \frac{x^2 + x + 2}{x-1} \rightarrow xy - y = x^2 + x + 2 \rightarrow x^2 + x + 2 - xy + y = 0$$

$$x^2 + x(1-y) + 2+y = 0 \quad \Delta \geq 0 \Rightarrow (1-y)^2 - 4(2+y) \geq 0$$

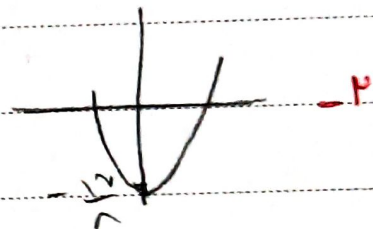
$$1+y^2-2y-4y-8 \geq 0 \rightarrow y^2-6y-7 \geq 0 \rightarrow \frac{-1}{2} \pm \frac{\sqrt{37}}{2}$$

$$R_f = (-\infty, -1] \cup [7, +\infty)$$

$$الف) y = x^2 - 2x + 1$$

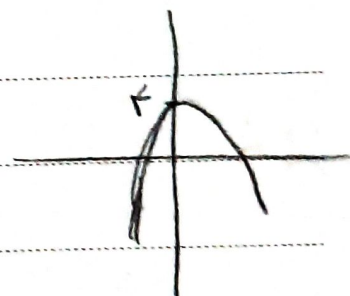
$$R_f = \left[-\frac{1}{4}, +\infty\right)$$

$$\begin{aligned} \Delta &= \left(\frac{-b}{2a}\right)^2 - \frac{b^2}{4a^2} \\ &= \frac{4}{4} = 1 \end{aligned}$$



$$ب) y = \sqrt{-x^2 + 4x - 4}$$

$$\begin{aligned} \Delta &= \left(\frac{-b}{2a}\right)^2 - \frac{b^2}{4a^2} \\ &= \frac{16}{4} = 4 \end{aligned}$$



$$R_f = \sqrt{[-4, 0]} = [0, 2]$$

$$ج) y = \sqrt{x^2 - 4x + 4} + 1$$

$$R_f = \sqrt{R} = [0, +\infty)$$

$$x^2 - 4x + 4 = (x-2)^2$$

$$د) y = \log$$

$$R_f = \mathbb{R}$$

$$x^2 - 4x + 4 \in \mathbb{R} \rightarrow \log(x^2 - 4x + 4) = \mathbb{R}$$

Subject.

Date.

$$1) y = \frac{ky+1}{y-k} \rightarrow y = - \frac{-1-1}{y-k} = \frac{1+y}{y-k}$$

$$\rightarrow y-k \neq 0 \rightarrow y \neq k \rightarrow y \neq \frac{k}{1}$$

$$R_f = \mathbb{R} - \left\{ \frac{k}{1} \right\}$$

$$2) y = \sqrt{\frac{ky+1}{y-k}} \quad y = - \frac{-1-1}{y-k} = \frac{1+y}{y-k} \rightarrow y-k \neq 0 \rightarrow y \neq k$$

$$R_f = \sqrt{\mathbb{R} - \left\{ \frac{k}{1} \right\}} = [0, \frac{k}{1}]$$

$$3) y = \frac{y^2+k}{\sqrt{y^2+k}} = \frac{y^2+k+1}{\sqrt{y^2+k}} = \sqrt{y^2+k} + \frac{1}{\sqrt{y^2+k}} \rightarrow \sqrt{y^2+k} \geq 0$$

$$\rightarrow R_f = [1, +\infty)$$

$$4) y = y^2 + \frac{1}{y^2+k}$$

$$y^2 \neq 0 \rightarrow y \neq 0 \rightarrow y = \frac{1}{k}$$

$$R_f = \left[\frac{1}{k}, +\infty \right)$$

$$y = \frac{ky^2+k}{y^2-1}$$

$$5) y^2 - y = ky^2 + k$$

$$\rightarrow y^2 - ky^2 - k + y \rightarrow y^2(y-k) = k+y \rightarrow y^2 = \frac{k+y}{y-k}$$

$$\rightarrow y = \pm \sqrt{\frac{k+y}{y-k}} \rightarrow \frac{k+y}{y-k} \geq 0 \rightarrow R_f = (-\infty, -k] \cup (k, +\infty)$$

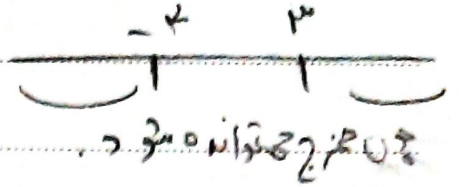
$$y \neq k$$

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Date.

$$P_{\infty} : x^P = 0 \rightarrow \frac{P(0) + K}{0-1} = -K$$

$$x^P = +\infty \rightarrow \frac{P(+\infty) + K}{+\infty-1} = P$$



$$R_f = (-\infty, -K] \cup (P, +\infty)$$

$$y = \frac{P \sin u - 1}{\sin u + P}$$

$$\text{مثال 1: } y \sin u + P y = P \sin u - 1$$

$$\rightarrow (P-y) \sin u = P y - 1 \rightarrow \sin u = \frac{P y - 1}{P-y}$$

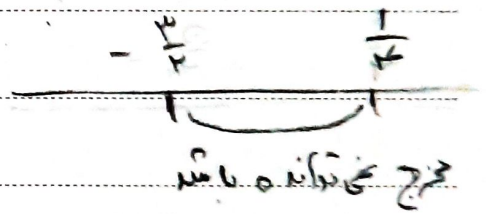
$$\rightarrow \frac{-1}{P-y} \leq \frac{P y - 1}{P-y} \leq \frac{1}{P-y}$$

$$y \in \left[\frac{P}{P+1}, 1 \right] \cup \left(-\infty, \frac{1}{P} \right] \cup (P, +\infty)$$

$$R_f \cap \Pi = \left[-\frac{P}{P+1}, \frac{1}{P} \right]$$

$$P_{\infty} : \sin u = -1 \rightarrow \frac{P(-1) - 1}{-1 + P} = \frac{-P}{P}$$

$$\sin u = 1 \rightarrow \frac{P(1) - 1}{1 + P} = \frac{1}{P}$$



$$R_f = \left[-\frac{P}{P+1}, \frac{1}{P} \right]$$

$$y = \frac{x^P - x + \frac{1}{P}}{x^P - x + 1} = \frac{(x - \frac{1}{P})^P}{(x - \frac{1}{P})^P + \frac{P}{P}}$$

$$\rightarrow y = \frac{z}{z + \frac{P}{P}} \xrightarrow{z = \frac{1}{P}} y = 0$$

$$R_f = [0, 1)$$

چون در نقطه از دامنه است هیچ وقت بد نمی شود

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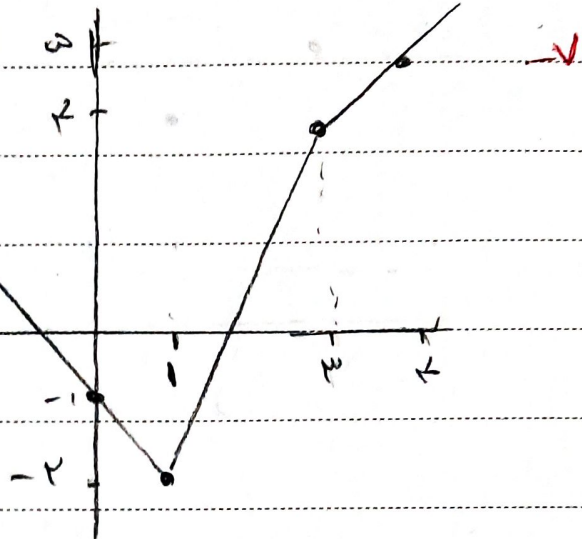
$$x^2 - x + 1 = 0 \rightarrow x = \frac{1 \pm \sqrt{1 - 4(1)}}{2} \rightarrow D < 0$$

پس یہ دو حقیقی اور متمیز نہیں ہوں گے۔

$$D_f = 1R$$

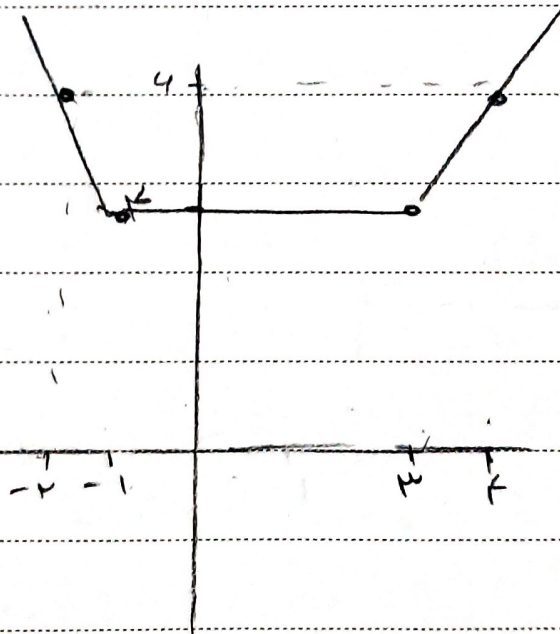
الف) $y = |2x - 1| - |x - 3|$

$$R_f = [-2, +\infty)$$



ب) $y = |x - 3| + |x + 1|$

$$R_f = [-2, +\infty)$$

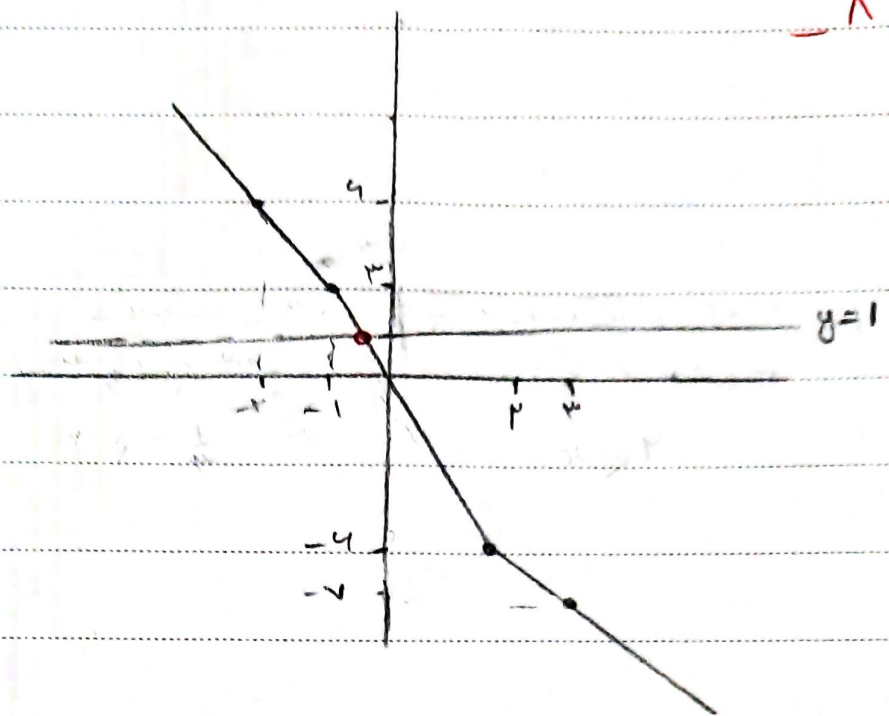


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$$|x - p| - |px + p| < 1$$

$\downarrow$ $\downarrow$
 p -1



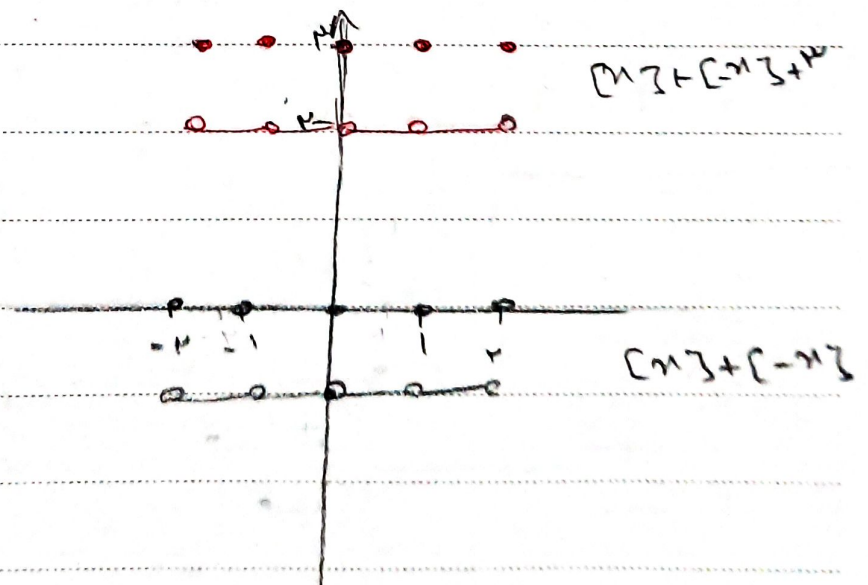
$$-1 \leq x \leq 1 \rightarrow p - x - px - p = -px$$

$$-px = 1 \rightarrow x = -\frac{1}{p}$$

$$|x - p| - |px + p| < 1 \rightarrow x \geq -\frac{1}{p}$$

$$\text{Case 1) } y = [x] + [p - x] = [x] + ([-x] + p) = [x] + [-x] + p$$

$$P_f = \{p, p\}$$



$$\hookrightarrow y = 3x - [2] \quad [-2, 2]$$

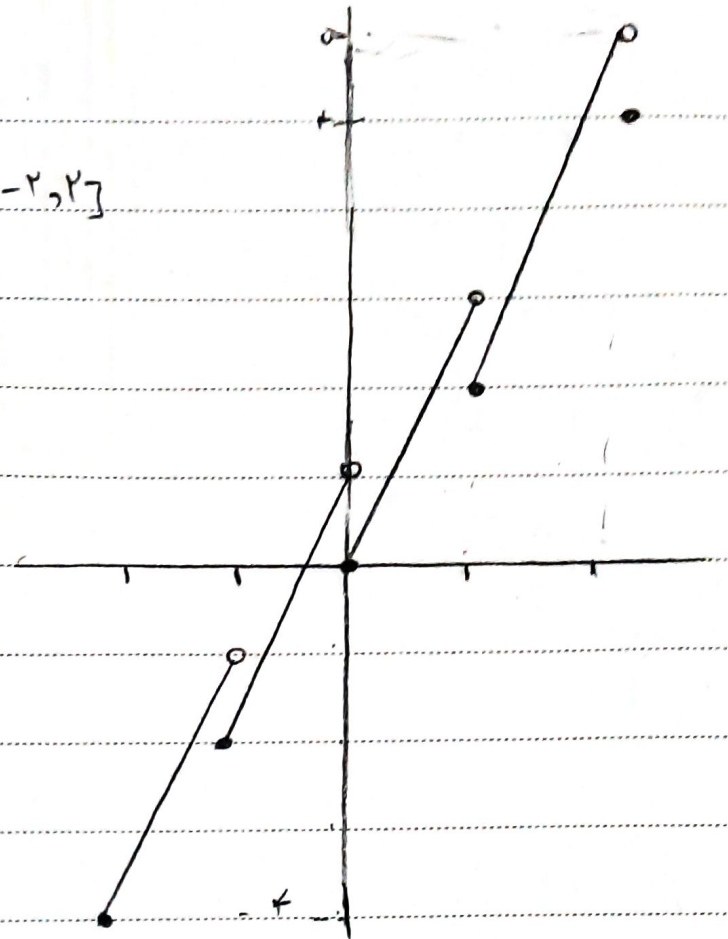
$$-2 \leq x < -1 \rightarrow y = 3x + 2$$

$$-1 \leq x < 0 \rightarrow y = 3x + 1$$

$$0 \leq x < 1 \rightarrow y = 3x$$

$$1 \leq x < 2 \rightarrow y = 3x - 1$$

$$x = 2 \rightarrow y = 3x - 2 = 4$$



$$R_f = [-2, \infty)$$

$$\text{الح) } y = \sin^2 x - \sin x + 1$$

$$\begin{array}{l} \downarrow 1 + 1 + 1 = 3 \\ \nearrow 1 - 1 + 1 = 1 \\ \downarrow \frac{1}{2} \end{array}$$

$$R_f = \left[\frac{1}{2}, 3\right]$$

$$\hookrightarrow y = \sin^2 x + \sin^2 x + 1$$

$$\begin{array}{l} \downarrow 0 + 0 + 1 = 1 \\ \nearrow 1 + 1 + 1 = 3 \\ \downarrow \frac{1}{2} \end{array}$$

عبارت منفی توان با س.

$$R_f = [1, 3]$$