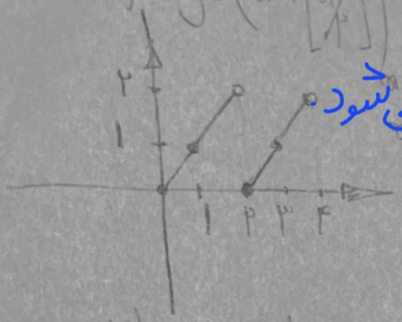
۱۶٫۷۵

به دلیل ننوشتن نام و نام خانوادگی نمرهٔ شما صفر در نظر گرفته می‌شود

-۱ ④

الف) $y=\left(x-۲\left[\frac{x}{۲}\right]\right)^۲ \longrightarrow$ $x-۲\left[\frac{x}{۲}\right] \to x=۱\to۱$, $x=۳\to۱$, $x=۲\to۰$, $x=۱۸\to۰$, $x=۱۹$, $۱۹-۲\left[\frac{۱۹}{۲}\right]=۱$



$x-۲\left[\frac{x}{۲}\right]\to[۰,۲)$ ✓ $y=\left(x-۲\left[\frac{x}{۲}\right]\right)^۲\to R_f=[۰,۴)$ ✓

ب) $-\sin x+\log y-۴=۰$ $\quad -۱\le\sin x\le۱$ ✓

$\log y=\sin x+۴$, $y>۰$

$۱۰^{۳}\le۱۰^{\sin x+۴}\le۱۰^{۵}\Rightarrow۱۰^{۳}\le y\le۱۰^{۵}\Rightarrow R_f=[۱۰^{۳},۱۰^{۵}]$ ✓ ✓

-۲ ④

$y=\sqrt{\frac{x^۲-۴}{x^۲-۹}}\to y\ge۰$ ✓

$y^۲=\frac{x^۲-۴}{x^۲-۹}\Rightarrow x^۲y^۲-۹y^۲=x^۲-۴$ $\quad x^۲(y^۲-۱)=-۴+۹y^۲$

$\Rightarrow \cancel{x^۲=\frac{-۴+۹y^۲}{y^۲-۱}}\Rightarrow x=\sqrt{\frac{-۴+۹y^۲}{y^۲-۱}}$ $\quad -۴+۹y^۲\to\pm\frac{۲}{۳}$, $y^۲-۱\to\pm۱$

		-۱		$-\frac{۲}{۳}$		$\frac{۲}{۳}$		۱	
$-۴+۹y^۲$	+		+	۰	−	۰	+		+
$y^۲-۱$	+	۰	−		−		−	۰	+
	+		−		+		−		+

$x^۲\to\min=۰\Rightarrow\sqrt{\frac{۴}{۹}}=\frac{۲}{۳}$ $\quad y\ge۰\Rightarrow R_f=[۰,\frac{۲}{۳}]\cup(۱,+\infty)$

$x^۲\to\max=+\infty\Rightarrow\sqrt{\frac{\infty}{\infty}}=۱$

$y\ge۰\Rightarrow R_f=[۰,\frac{۲}{۳}]\cup(۱,+\infty)$ ✓

$a=۰$, $b=\frac{۲}{۳}$, $c=۱\Rightarrow a+b+c=\frac{۵}{۳}$ ✓

-۳

الف) $f(x)=x+\frac{۱}{x}+۲\sqrt{x+\frac{۱}{x}}$ ✓ $x+\frac{۱}{x}\ge۲$, $-x+\frac{۱}{x}\le-۲$ $\quad x+\frac{۱}{x}\ge۰\to x$ $\Rightarrow ۲+۲\sqrt{۲}\Rightarrow R_f=[۲+۲\sqrt{۲},+\infty)$ ✓

ب) $y=(۳-\sin x)(۱+\sin x)=۳+۳\sin x-\sin x-\sin^۲x=۳+۲\sin x-\sin^۲x$ $\quad -۱\le\sin x\le۱$ ✓ ②

$\to\max\ ۳+۲-۱=۴$

$\min\ ۳-۲-۱=۰\Rightarrow R_f=[۰,۴]$ ✓

✓ $-\frac{b}{۲a}=-\frac{۲}{-۲}=۱$

-۴ ⑤

دنبالهٔ حسابی

$f(x)=-\frac{۱}{۳}x+۱۰\to x=۱\Rightarrow-\frac{۲۹}{۳}$ $\quad x=۲\Rightarrow-\frac{۲۸}{۳}$ $\quad x=۳\Rightarrow-\frac{۲۷}{۳}$ $\quad\dots\ x=۹\Rightarrow-\frac{۲۱}{۳}$

$R_f=\left\{\frac{۲۱}{۳},\frac{۲۲}{۳},\dots,\frac{۲۹}{۳}\right\}$ $\to ۹\times-۱۰+(-۹+-۸+\dots+-۱)=\frac{-۱۸۰-۴۵}{۲}=\frac{-۲۲۵}{۳}=-۷۵$

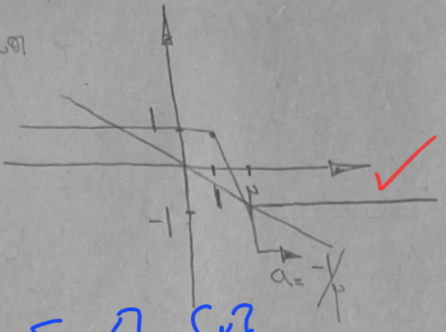
$S=\frac{n}{۲}(a_۱+a_n)\to S=\frac{۹}{۲}\left(\frac{۲۱}{۳}+\frac{۲۹}{۳}\right)=\boxed{۷۵}$ $\to\frac{۹۰}{۲}=۴۵$

$f(x)=\sqrt{ax^۲+bx+c}$ $\quad D_f=[۲,+\infty)\Rightarrow a=۰$ ✓ $\quad ۲b+c=۰$, $۹a+۳b+c=۱\Rightarrow b=۱$, $c=-۲$ ✓ $\quad g(x)=\sqrt{x^۲+۴}$, ±۲ $\quad R_g=(-\infty,-۲]\cup[\frac{۵}{۲},+\infty)$ ④

$R_f=[۲,+\infty)$ ✓ $\Rightarrow\frac{-b^۲+۴ac}{۴a}=۲$ $\quad ۸a=-b^۲+۴ac$

-۶

$|x-2|-|x-1| = kx$



$kx \geq -\frac{1}{2}x \quad k=-\frac{1}{2}$ ✓

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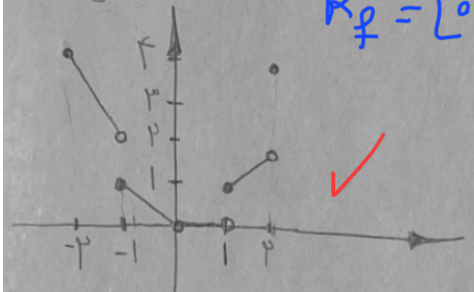
-۷

الف) $y = x[x]$

$R_f = [0,4]-\{2\}$



$x=0 \Rightarrow 0 \leq x < 1 \rightarrow x \times 0 = 0$

$x=1 \Rightarrow 1 \leq x < 2 \rightarrow x \times 1 = x \quad 1/9 \times 1 = 1/9$ ✓

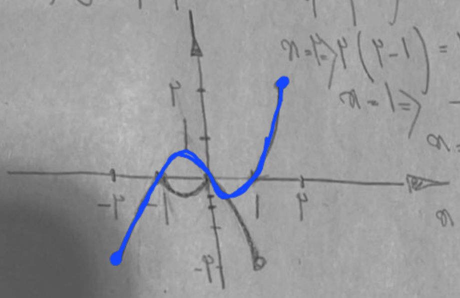
$x=2 \Rightarrow 4$

$x=-1 \Rightarrow -1 \leq x < 0 \rightarrow -1 \times x = -x \quad -0/99 \times -1 = 0/99$

$x=-2 \Rightarrow -2 \leq x < -1 \rightarrow -2 \times x = -2x \quad -1/9 \times -2 = 3/8 \quad -1/1 \times -2 = +2/2$

۰/۷۵

ب) $y = x|x| - x = x(|x|-1)$



$x=2 \Rightarrow 2(2-1)=2$

$x=1 \Rightarrow -1-1=-2$

$x=-1 \Rightarrow -1+1=0$

$x=-\frac{1}{2} \Rightarrow -\frac{1}{2}(+\frac{1}{2})+\frac{1}{2} = -\frac{1}{2}$

$-\frac{1}{4}$

$x=-2 \Rightarrow -4+2=-2$

$x=-\frac{3}{2} \Rightarrow -\frac{3}{2}(+\frac{3}{2})+\frac{3}{2} = -\frac{9}{4}+\frac{6}{4} = -\frac{3}{4}$

$x=0$

$x=\pm 1$

	-1	0	1	
	$-x(+x-1)$ $-x^2+x$	$-x(-x-1)$ $+x^2+x$	$x(-x-1)$ $-x^2-x$	x^2-x
$\lvert x\rvert-1$	+	-	-	+
x	-	-	+	+
	-	+	-	+

$$\begin{array}{c|c} & 0 \\ \hline -x^2-x & x^2-x \end{array}$$

$R_f = [-2,2]$

$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x}$

$x \neq -1, 0$

$y = \frac{x+1+x+1}{x^2+x} = \frac{2x+2}{x^2+x}$ ✓ $\quad yx^2+yx = 2x+2 \quad yx^2-2x+yx-2=0$ -۸

۲

$\Rightarrow yx^2+(y-2)x-2=0$ دلتا $(y-2)^2+8y>0 \quad y^2+4+8y-4y>0 \quad y^2+4y+4>0$

$\frac{2\cancel{(x+1)}}{x\cancel{(x+1)}} = \frac{2}{x} \quad y=\frac{2}{x} \rightarrow y \neq 0$ ✓ $\quad (y+2)^2>0 \quad y \neq -2$ ✓

$y \neq \frac{+2}{-1} = -2 \quad a+b = -2+0 = -2$ ✓

$y = \frac{2(x+1)}{x(x+1)} = \frac{2}{x}$

$f(x) = \frac{x-4\sqrt{x}+3}{x-2\sqrt{x}+1}$

$\sqrt{x}=t \quad x \geq 0$

$y = \frac{t^2-4t+3}{t^2-2t+1} = \frac{(t-3)\cancel{(t-1)}}{\cancel{(t-1)}(t-1)} = \frac{t-3}{t-1} \rightarrow \frac{y-3}{y-1}$ ✓ -۹

$t \neq 1$

$yt-y=t-3 \Rightarrow yt-t=y-3 \quad t(y-1)=y-3 \Rightarrow t=\frac{y-3}{y-1}$

۲

$y \neq 1 \quad \frac{y-3}{y-1} \geq 0 \quad \begin{array}{c|c|c} + & 1 \quad - & 3 \quad + \end{array} \Rightarrow R_f = (-\infty, +1) \cup [3, +\infty)$ ✓

-۱۰

$$f(x)=\frac{x^{۳}+۲x^{۲}-x-۲}{x^{۲}-۱}=\frac{x^{۲}(x+۲)-(x+۲)}{x^{۲}-۱}=\frac{\cancel{(x^{۲}-۱)}(x+۲)}{\cancel{x^{۲}-۱}}\Rightarrow \begin{matrix}۱+۲=۳\\ -۱+۲=۱\end{matrix}\Rightarrow ۳+۱=۴$$ ✓

$$x^{۲}-۱ \longrightarrow x\neq \pm۱$$ ✓

۲