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سوال ۱

$$\text{الف) } y = (x - 2[\frac{x}{4}])^2 \Rightarrow y = (2(\frac{x}{4} - [\frac{x}{4}]))^2 = 4(\frac{x}{4} - [\frac{x}{4}])^2$$

$$\cdot \ll \frac{x}{4} - [\frac{x}{4}] < 1 \rightarrow \cdot \ll 4(\frac{x}{4} - [\frac{x}{4}])^2 < 4 \quad R_f = [0, 4]$$

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ب) $-\sin x + 1 \cdot \log y - f = 0$

$\log y = \sin x + f$

$-1 \leq \sin x \leq 1 \rightarrow -1 \leq \sin x + f \leq 5 \Rightarrow -1 \leq \log y \leq 5 \Rightarrow 10^{-1} \leq y \leq 10^5 \Rightarrow R_f = [10^{-1}, 10^5]$

$f(x) = \sqrt{\frac{x^2 - 4}{x^2 - 9}}$
 $a + b + c = ?$

$R_f = [a, b] \cup (c, +\infty)$
 $R_f = [0, \frac{4}{3}] \cup (1, +\infty)$
 $y^2 = \frac{x^2 - 4}{x^2 - 9} \Rightarrow x^2 y^2 - 4y^2 = x^2 - 9 \Rightarrow x^2 y^2 - x^2 = 4y^2 - 9 \Rightarrow x^2(y^2 - 1) = 4y^2 - 9$

$\Rightarrow x^2 = \frac{4y^2 - 9}{y^2 - 1} \Rightarrow x = \sqrt{\frac{4y^2 - 9}{y^2 - 1}} \Rightarrow \frac{4y^2 - 9}{y^2 - 1} \geq 0$

$y = \pm \frac{3}{2}$
 $y = \pm 1$

$\Rightarrow R_f = [-\frac{3}{2}, \frac{3}{2}] \cup (1, +\infty) \Rightarrow a + b + c = -\frac{3}{2} + \frac{3}{2} + 1 = 1$

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}}$

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$(\sqrt{x + \frac{1}{x}} + 1)^2 - 1$

$x=1$
 $\Rightarrow$
 $\frac{1}{x}$
 $\sqrt{x + \frac{1}{x}}$

$(\sqrt{2} + 1)^2 - 1 \Rightarrow 2 + 1 + 2\sqrt{2} - 1 = 2 + 2\sqrt{2} \Rightarrow R_f = [2 + 2\sqrt{2}, +\infty)$

سوال ۳) می توانیم به فرم $ax^2 + bx + c$ هم حل کنیم.

ب) $y = (3 - \sin x)(1 + \sin x) = 3 + 3\sin x - \sin x - \sin^2 x \Rightarrow y = -\sin^2 x + 2\sin x + 3$
 $\max \rightarrow -1 + 2 + 3 = 4$
 $\min \rightarrow -1 - 2 + 3 = 0$

$R_f = [0, 4]$

$f(x) = -\frac{1}{10}x + 10$

$D_f = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$R_f = ?$

$x=1 \Rightarrow y = 10 - \frac{1}{10} = \frac{99}{10}$

$x=2 \Rightarrow y = 10 - \frac{1}{5} = \frac{19}{5}$

$x=3 \Rightarrow y = 10 - \frac{1}{3} = \frac{29}{3}$

$x=4 \Rightarrow y = 10 - \frac{1}{4} = \frac{39}{4}$

$x=5 \Rightarrow y = 10 - \frac{1}{5} = \frac{49}{5}$

$x=6 \Rightarrow y = 10 - \frac{1}{6} = \frac{59}{6}$

$x=7 \Rightarrow y = 10 - \frac{1}{7} = \frac{69}{7}$

$x=8 \Rightarrow y = 10 - \frac{1}{8} = \frac{79}{8}$

$x=9 \Rightarrow y = 10 - \frac{1}{9} = \frac{89}{9}$

$\Rightarrow R_f = \{\frac{99}{10}, \frac{19}{5}, \frac{29}{3}, \frac{39}{4}, \frac{49}{5}, \frac{59}{6}, \frac{69}{7}, \frac{79}{8}, \frac{89}{9}\}$

$f(x) = \sqrt{ax^2 + bx + c}$

$D_f = [2, +\infty)$

$f(2) = 1$

$g(x) = \sqrt{bx^2 + ax - 4c}$

$R_g = ?$

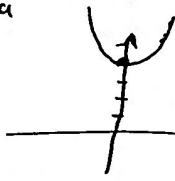
سوال ۵

$\Rightarrow \begin{cases} 4a + 3b + c = 1 \\ 4a + 3 - 10a + c = 1 \\ 4a + c = -2 \end{cases}$

$\begin{cases} 4a + c = -2 \\ -4a + c = -2 \end{cases} \rightarrow \begin{cases} 4a + c = -2 \\ -4a + c = -2 \end{cases}$

$\Rightarrow g(x) = \sqrt{x^2 + 4}$

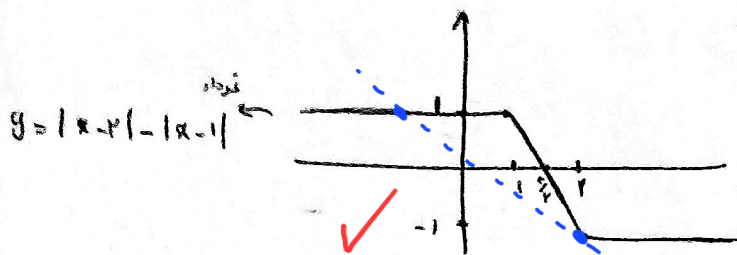
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$R = [2, +\infty)$

مقدار K نظری که این معادله دو جواب داشته باشد $|x-2| - |x-1| = Kx$

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$x=1 \Rightarrow y=1$
 $x=2 \Rightarrow y=-1$

برای آنکه ۲ جواب داشته باشد $y = Kx$

باید از نقطه $(\frac{2}{-1})$ عبور کند $K = \frac{1}{2}$

الف) $y = x[x]$

$-2 \leq x < -1 \Rightarrow y = -2x$

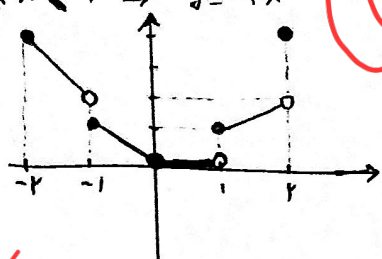
$-1 \leq x < 0 \Rightarrow y = -x$

$0 \leq x < 1 \Rightarrow y = 0$

$1 \leq x < 2 \Rightarrow y = x$

$x=2 \Rightarrow y=2x$

$\Rightarrow y=4$



$R_f = [0, 4] - \{2\}$

ب) $y = x|x| - x$

$\frac{-x^2-x}{x^2-x}$

سوال ۲) ۳ مرحله $[-2, 2]$

$-2 \leq x < -1 \Rightarrow y = -x^2 - x$

$-1 \leq x < 0 \Rightarrow y = -x^2 - x$

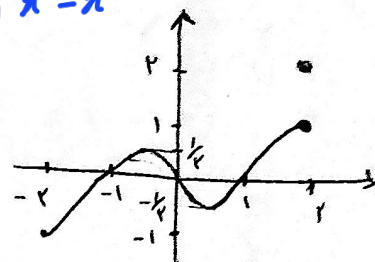
$0 \leq x < 1 \Rightarrow y = x^2 - x$

$1 \leq x < 2 \Rightarrow y = x^2 - x$

$x=2 \Rightarrow y=4-2=2$

$R_f = [-2, 2]$

$R_f = [-1, 1] \cup \{2\}$



$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x}$

$R_f = \mathbb{R} - \{a, b\}$

$a+b=?$

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سوال ۳

$y = \frac{x+1+x+1}{x(x+1)} = \frac{2x+2}{x(x+1)} = \frac{2(x+1)}{x(x+1)} \Rightarrow R_f = \mathbb{R} - \{0, -1\} \Rightarrow x=0 \Rightarrow y=\infty$
 $x=-1 \Rightarrow y = \frac{2}{-1} = -2$
 $\Rightarrow R_f = \mathbb{R} - \{0, -2\}$

صیغله منفرجه شود

$a+b = 0-2 = -2$

$f(x) = \frac{x - x\sqrt{x} + \frac{x^2}{x-1}}{x - x\sqrt{x} + 1} = \frac{(\sqrt{x}-1)(\sqrt{x}-1)}{(\sqrt{x}-1)^2} = \frac{\sqrt{x}-1}{\sqrt{x}-1}$

سوال ۴

$\frac{aD+b}{cD+d}$

$\sqrt{x} = D \rightarrow \begin{cases} f(\infty) = 1 \\ f(1) = 3 \end{cases}$

$R_f = (-\infty, 1) \cup [3, +\infty)$

$f(x) = \frac{x^3 + x^2 - x - 2}{x^2 - 1}$

$R_f = \mathbb{R} - \{x, y\}$

$x+y=?$

سوال ۵

$f(x) = \frac{x^3(x+2) - (x+2)}{(x^2-1)} = \frac{(x^2-1)(x+2)}{(x^2-1)} \Rightarrow y = x+2 \rightarrow R_f = \mathbb{R} - \{\pm 1\}$

$\begin{cases} x=+1 \Rightarrow y=3 \\ x=-1 \Rightarrow y=1 \end{cases}$

$\Rightarrow R_f = \mathbb{R} - \{3, 1\} \leadsto x, y = 3, 1 \Rightarrow x+y = 3+1 = 4$