

سوال 1

$$\text{الف) } y = (x - 2 \lfloor \frac{x}{2} \rfloor)^2 \Rightarrow y = (2(\frac{x}{2} - \lfloor \frac{x}{2} \rfloor))^2 = 4(\frac{x}{2} - \lfloor \frac{x}{2} \rfloor)^2$$

ب) $-\sin x + 1 \cdot \log y - f = 0$

$$\log y = \sin x + f$$

$$-1 \leq \sin x \leq 1 \rightarrow -1 \leq \sin x + f \leq 1 \Rightarrow -1 \leq \log y \leq 1 \Rightarrow 10^{-1} \leq y \leq 10^1 \Rightarrow R_f = [10^{-1}, 10^1]$$

سوال 2

$$f(x) = \sqrt{\frac{x^2 - 4}{x^2 - 9}}$$

$$R_f = [a, b] \cup (c, +\infty)$$

$$a + b + c = ? \rightarrow y^2 = \frac{x^2 - 4}{x^2 - 9} \Rightarrow x^2 y^2 - 9y^2 = x^2 - 4 \Rightarrow x^2 y^2 - x^2 = 9y^2 - 4 \Rightarrow x^2(y^2 - 1) = 9y^2 - 4$$

$$\Rightarrow x^2 = \frac{9y^2 - 4}{y^2 - 1} \Rightarrow x = \sqrt{\frac{9y^2 - 4}{y^2 - 1}} \Rightarrow \frac{9y^2 - 4}{y^2 - 1} \geq 0$$

$$y = \pm \frac{2}{3} \quad \begin{array}{c} -1 \quad -\frac{2}{3} \quad \frac{2}{3} \quad 1 \\ + \quad - \quad + \quad - \quad + \\ \hline \frac{1}{3} \end{array}$$

$$\Rightarrow R_f = [-\frac{2}{3}, \frac{2}{3}] \cup (1, +\infty) \Rightarrow a + b + c = -\frac{2}{3} + \frac{2}{3} + 1 = 1$$

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}}$

$$(\sqrt{x + \frac{1}{x}} + 1)^2 - 1$$

$$x = 1$$

$$\Rightarrow (\sqrt{2} + 1)^2 - 1 \Rightarrow 2 + 1 + 2\sqrt{2} - 1 = 2 + 2\sqrt{2} \Rightarrow R_f = [2 + 2\sqrt{2}, +\infty)$$

سوال 3

ب) $y = (3 - \sin x)(1 + \sin x)$

$$= 3 + 3\sin x - \sin x - \sin^2 x \Rightarrow y = -\sin^2 x + 2\sin x + 3$$

$$\begin{array}{l} \max \rightarrow -1 + 2 + 3 = 4 \\ \min \rightarrow -1 - 2 + 3 = 0 \end{array}$$

$$R_f = [0, 4] \quad \leftarrow \frac{r}{-r} = 1 \rightarrow -1 + 2 + 3 = 4$$

$$f(x) = -\frac{1}{x} x + 10$$

$$D_f = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$R_f = ?$$

سوال 4

$$x = 1 \Rightarrow y = 10 - \frac{1}{1} = 9$$

$$x = 2 \Rightarrow y = 10 - \frac{1}{2} = \frac{19}{2}$$

$$x = 3 \Rightarrow y = 10 - \frac{1}{3} = \frac{29}{3}$$

$$x = 4 \Rightarrow y = 10 - \frac{1}{4} = \frac{39}{4}$$

$$x = 5 \Rightarrow y = 10 - \frac{1}{5} = \frac{49}{5}$$

$$x = 6 \Rightarrow y = 10 - \frac{1}{6} = \frac{59}{6}$$

$$x = 7 \Rightarrow y = 10 - \frac{1}{7} = \frac{69}{7}$$

$$x = 8 \Rightarrow y = 10 - \frac{1}{8} = \frac{79}{8}$$

$$x = 9 \Rightarrow y = 10 - \frac{1}{9} = \frac{89}{9}$$

$$\Rightarrow R_f = \{9, \frac{19}{2}, \frac{29}{3}, \frac{39}{4}, \frac{49}{5}, \frac{59}{6}, \frac{69}{7}, \frac{79}{8}, \frac{89}{9}\}$$

$$f(x) = \sqrt{ax^2 + bx + c}$$

$$D_f = [2, +\infty)$$

$$f(2) = 1$$

$$g(x) = \sqrt{bx^2 + ax - 4c}$$

$$R_g = ?$$

سوال 5

$$\Rightarrow \begin{cases} 4a + 3b + c = 1 \\ 4a + 3 - 15a + c = 1 \end{cases} \Rightarrow \begin{cases} 4a + 3b + c = 1 \\ 4a + c = -2 \end{cases}$$

$$+ 4a + 3b + c = 0$$

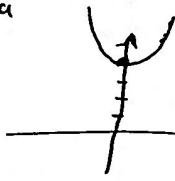
$$4a + c = -2$$

$$5a + b = 1 \Rightarrow b = 1 - 5a$$

$$4a + 3 - 10a + c = 0 \Rightarrow -4a + c = -3$$

$$\begin{cases} 4a + c = -2 \\ -4a + c = -3 \end{cases} \rightarrow \begin{cases} 4a + c = -2 \\ -4a + c = -3 \end{cases}$$

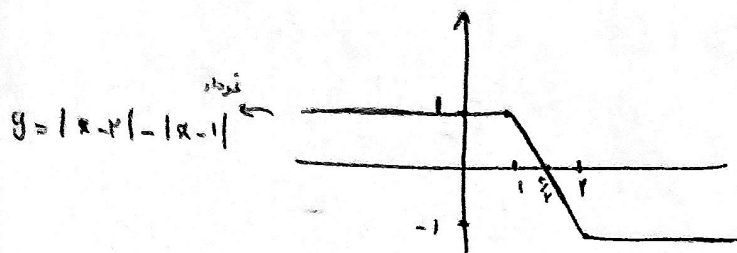
$$\Rightarrow g(x) = \sqrt{ax^2 + 4} \rightarrow \text{این تابع را می توانیم به صورت } x^2 + 4 \text{ بنویسیم}$$



$$R = [2, +\infty) \xrightarrow{\text{برای } x=0} = [2, +\infty)$$

$$\begin{aligned} 4c = -4 &\Rightarrow c = -1 \Rightarrow a = 0 \\ b = 1 - 5(0) &= 1 \end{aligned}$$

مقدار K به طوری که این معادله دو جواب داشته باشد $|x-2| - |x-1| = Kx$ $\Rightarrow$



$x=1 \Rightarrow y=1$

$x=2 \Rightarrow y=-1$

الف) $y = x[x]$ $-2 \leq x < -1 \Rightarrow y = -2x$

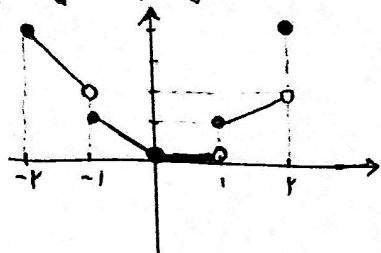
$-1 \leq x < 0 \Rightarrow y = -x$

$0 \leq x < 1 \Rightarrow y = 0$

$1 \leq x < 2 \Rightarrow y = x$

$x=2 \Rightarrow y=2x$

$\Rightarrow y=4$



$R_f = [0, 2] - \{2\}$

ب) $y = x|x| - x$

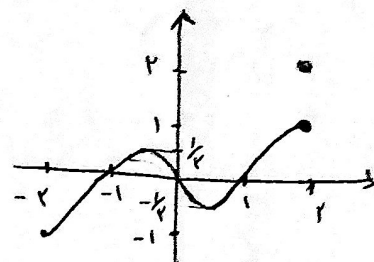
$-2 \leq x < -1 \Rightarrow y = -x^2 - x$

$-1 \leq x < 0 \Rightarrow y = -x^2 - x$

$0 \leq x < 1 \Rightarrow y = x^2 - x$

$1 \leq x < 2 \Rightarrow y = x^2 - x$

$x=2 \Rightarrow y=4-2=2$



$R_f = [-1, 1] \cup \{2\}$

$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x}$
 $\downarrow \quad \downarrow \quad \downarrow$
 $x(x+1) \quad x(x) \quad x(x+1)$

$R_f = \mathbb{R} - \{a, b\}$

$a+b=?$

سوال ۸

$y = \frac{x+1+x+1}{x(x+1)} = \frac{2x+2}{x(x+1)} = \frac{2(x+1)}{x(x+1)} \Rightarrow R_f = \mathbb{R} - \{0, -1\} \Rightarrow x=0 \Rightarrow y=\infty$
 $x=-1 \Rightarrow y = \frac{2}{-1} = -2$
 $\Rightarrow R_f = \mathbb{R} - \{0, -2\}$
 $a+b = 0-2 = -2$

$f(x) = \frac{x - x\sqrt{x+1} + \frac{x^2}{x+1}}{x - x\sqrt{x+1} + 1}$
 $(\sqrt{x}-1)^2$

سوال ۹

$f(x) = \frac{x^3 + x^2 - x - 2}{x^2 - 1}$

$R_f = \mathbb{R} - \{x, y\}$

$x+y=?$

$\hookrightarrow \frac{x^3(x+1) - (x+2)}{(x^2-1)} = \frac{(x^2-1)(x+2)}{(x^2-1)} \Rightarrow y = x+2 \rightarrow R_f = \mathbb{R} - \{\pm 1\}$

ریشه های خارج

سوال ۱۰

$\left. \begin{array}{l} x=+1 \Rightarrow y=3 \\ x=-1 \Rightarrow y=1 \end{array} \right\} \Rightarrow R_f = \mathbb{R} - \{3, 1\} \leadsto x, y = 3, 1 \Rightarrow x+y = 3+1 = \boxed{4}$