نام و نام خانوادگی : زهرا ارشد   پایه یازدهم رشته ریاضی شماره ۴   کلاس ۱۱۰

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الف) $y=(x-2[\frac{x}{2}])^2$

$(2(\frac{x}{2}-[\frac{x}{2}]))^2$ $x-[x]\in[0,1)$

$0\le \frac{x}{2}-[\frac{x}{2}]<1 \xrightarrow{\times 2} 0\le 2(\frac{x}{2}-[\frac{x}{2}])<2$

$\xrightarrow{\text{توان ۲}} 0\le y<4 \Rightarrow R_f=[0,4)$

ب) $-\sin x+\log y-4=0$

$\log y=\sin x+4$

$-1\le\sin x\le 1 \to 3\le \sin x+4\le 5$

$\to 3\le\log y\le 5 \to 10^3\le y\le 10^5$

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$f(x)=\sqrt{\frac{x^2-4}{x^2-9}} \xrightarrow{R_f} [a,b]\cup(c,+\infty)$ $a+b+c=?$

$y=\sqrt{\frac{x^2-4}{x^2-1}} \longrightarrow y^2=\frac{x^2-4}{x^2-9} \to y^2x^2-9y^2=x^2-4$

$\to y^2x^2=9y^2-4 \Rightarrow x^2(y^2-1)=9y^2-4 \Rightarrow x^2=\frac{9y^2-4}{y^2-1}$

$x=\sqrt{\frac{9y^2-4}{y^2-1}} \xrightarrow{\text{زیر رادیکال}\ge 0}$ (I) $\frac{9y^2-4}{y^2-1}\ge 0 \longrightarrow$ $-1 \quad -\frac{2}{3} \quad \frac{2}{3} \quad 1$

II $y\ge 0$ $\xrightarrow[I\cap II]{\text{اشتراک}} [0,\frac{2}{3}]\cup(1,+\infty) \Rightarrow a+b+c=\boxed{\frac{5}{3}}$ جواب

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الف) $f(x)=x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}}$

$(\sqrt{x+\frac{1}{x}}+1)^2-1$

$\xrightarrow{\text{کمترین مقدار}} (\sqrt{2}+1)^2-1 \longrightarrow 2+1+2\sqrt{2}-1$

$\longrightarrow 2+2\sqrt{2}$

$[2+2\sqrt{2}, +\infty)$

ب) $y=(3-\sin x)(1+\sin x)$

$3+3\sin x-\sin x-\sin^2 x$

$y=-\sin^2 x+2\sin x+3$

$\xrightarrow{max} 1 \Rightarrow -1+2+3=4 \Rightarrow R=[0,4]$

$\xrightarrow{min} -1 \Rightarrow -1-2+3=0$

$\frac{-b}{2a} \to \frac{-2}{-2}=1 \to -1+2+3=4$

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اعداد طبیعی یک رقمی $\in \{1,2,3,4,5,6,7,8,9\}$

$f(x)=-\frac{1}{3}x+10$

$\xrightarrow{x=1} 10-\frac{1}{3}=\frac{29}{3}$

$\xrightarrow{x=2} 10-\frac{2}{3}=\frac{28}{3}$

$\xrightarrow{x=3} 10-1=9$

$\xrightarrow{x=4} 10-\frac{4}{3}=\frac{26}{3}$

$\xrightarrow{x=5} 10-\frac{5}{3}=\frac{25}{3}$

$\xrightarrow{x=6} 10-2=8$

$\frac{23}{3}=10-\frac{7}{3} \xleftarrow{x=7}$

$\frac{22}{3}=10-\frac{8}{3} \xleftarrow{x=8}$

$7=10-\frac{9}{3} \xleftarrow{x=9}$

$\Rightarrow R_f=\{\frac{29}{3},\frac{28}{3},9,\frac{26}{3},\frac{25}{3},8,\frac{23}{3},\frac{22}{3},7\}$

$\frac{22}{3}+\frac{29}{3}+\frac{28}{3}+9+\frac{26}{3}+\frac{25}{3}+8+\frac{23}{3}+7$

$\Downarrow$

$\boxed{75}$

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$f(x)=\sqrt{\underset{0}{a}x^2+\underset{1}{b}x+\underset{-2}{c}} \xrightarrow{[2,+\infty)\ \text{دارد}}$

$f(3)=1$

$g(x)=\sqrt{bx^2+ax-2c}=?$

$9a+3b+c=1 \to 9a+3-15a+c=1$ $\boxed{-6a+c=-2}$

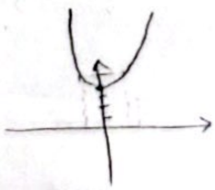
$4a+2b+c=0$ $4a+2-10a+c=0$

$5a+b=1 \to b=1-5a$ $\boxed{-6a+c=-2}$

$\Rightarrow \begin{cases} 6a+c=-2 \\ -6a+c=-2 \end{cases}$ $\xrightarrow{c=-2} a=0$

$2c=-4 \to \boxed{c=-2}$

$\sqrt{x^2+4}$

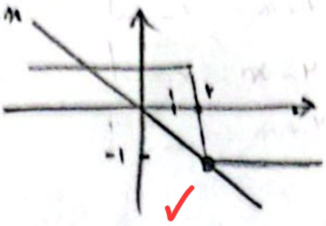


$\xrightarrow{1}$ بعد جذر در نظر گرفتن

$\longrightarrow [4,+\infty) \xrightarrow{\text{چون زیر رادیکال}} [2,+\infty)$

$|x-2| - |x-1| = kx$

گرافش را رسم میکنیم



$y = kx \xrightarrow{(2,-1)} -1 = 2k \rightarrow \boxed{k = -1/2}$ ۲

الف) $y = x[x]$ ۱٫۵

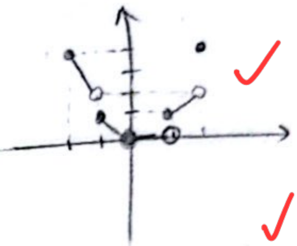
$-2 \leq x < -1 \rightarrow -2x$

$-1 \leq x < 0 \rightarrow -x$

$0 \leq x < 1 \rightarrow 0$

$1 \leq x < 2 \rightarrow x$

$x = 2 \rightarrow 2x$



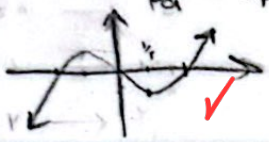
$R = [0, 4] - \{2\}$

ب) $y = x|x| - x$

$\begin{array}{c|c} -x^2 - x & x^2 - x \end{array}$ (0)

$\rightarrow \dfrac{-b}{2a} = 1/2 \quad y = -1/4$

$\rightarrow \dfrac{-b}{2a} = -1/2 \rightarrow y = 1/4$



$R_f = \mathbb{R}$

$R_f = [-2, 2]$

$y = \dfrac{1}{x} + \dfrac{1}{x+1} + \dfrac{1}{x^2+x} \longrightarrow R_f = \mathbb{R} - \{a, b\} \longrightarrow a + b = ?$

$y = \dfrac{x + x + 2}{x^2 + x} = \dfrac{2x + 2}{x^2 + x} \longrightarrow \dfrac{2(x+1)}{x(x+1)} \xrightarrow{x \neq -1} \dfrac{2}{x} = y$ ۲

$x = -1 \longrightarrow \dfrac{2}{-1} = -2$ مجانب $= \dfrac{0}{x} = 0$ $a = -2$ $b = 0$

$a + b = -2 + 0 = -2$

$\Longrightarrow y = \dfrac{\sqrt{x} - 3}{\sqrt{x} - 1} \longrightarrow y\sqrt{x} - y = \sqrt{x} - 3$ $\quad y = \dfrac{(\sqrt{x} - 3)(\sqrt{x} - 1)}{(\sqrt{x} - 1)(\sqrt{x} - 1)} \xrightarrow[x \neq 1]{\text{اگر}}$

$y\sqrt{x} - \sqrt{x} = y - 3 \longrightarrow \sqrt{x} = \dfrac{y-3}{y-1} \Longrightarrow \dfrac{y-3}{y-1} \geq 0 \rightarrow$ $\begin{array}{ccc} 1 & & 3 \\ + & - & + \end{array}$

$R_f = (-\infty, 1] \cup [3, +\infty)$ ۱٫۷۵

$R_f = (-\infty, 1) \cup [3, +\infty)$

$\dfrac{\cancel{(x+1)}(x^2 + x - 2)}{(x-1)\cancel{(x+1)}} \longrightarrow \dfrac{(x+2)\cancel{(x-1)}}{\cancel{(x-1)}} = x + 2 \xrightarrow{x=1} 3$ ۲

$\rightarrow x \neq \pm 1$ $\qquad \xrightarrow{x \neq 1} 1$

$\longrightarrow \boxed{x = 4}$