

الف) $y = (x - 2[\frac{x}{2}])^2 = 4(\frac{x}{2} - [\frac{x}{2}])^2 \rightarrow [0, 1) \xrightarrow{200\%} [0, 1) \cdot x^2 \rightarrow [0, 4) = R$

ب) $-\sin x + \log y - t = 0 \rightarrow \log y - t = \sin x \rightarrow -1 \leq \log y - t \leq 1$
 $3 \leq \log y \leq 5 \rightarrow y \geq 10^3, y \leq 10^5 \Rightarrow R = [10^3, 10^5]$

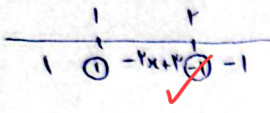
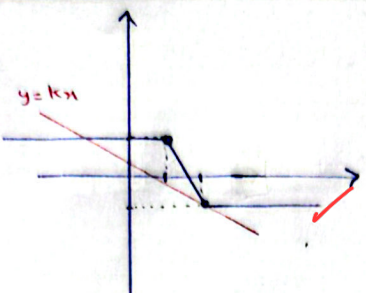
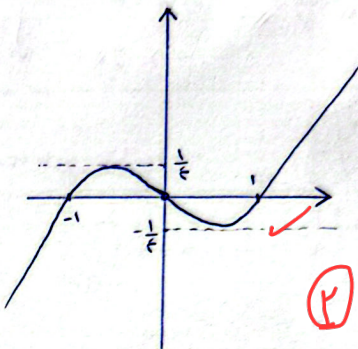
$f(x) = \sqrt{\frac{x^2 - 4}{x^2 - 9}} \rightarrow y \geq 0$ (سبب اول) $\rightarrow y^2 = \frac{x^2 - 4}{x^2 - 9} \rightarrow y^2 x^2 - 4y^2 = x^2 - 4 \rightarrow y^2 x^2 - x^2 = 4y^2 - 4$
 $x^2 = \frac{4y^2 - 4}{y^2 - 1} \rightarrow x = \pm \sqrt{\frac{4y^2 - 4}{y^2 - 1}} \rightarrow \frac{4y^2 - 4}{y^2 - 1} \geq 0 \rightarrow \frac{-1}{\frac{1}{y} - 1} + \frac{\frac{4}{y}}{\frac{1}{y} - 1} + \frac{1}{\frac{1}{y} - 1} \rightarrow R = [0, \frac{2}{3}] \cup (1, +\infty)$
 $[0, \frac{2}{3}] \cup (1, +\infty) = [a, b] \cup (c, +\infty) \rightarrow \begin{cases} a=0 \\ b=\frac{2}{3} \\ c=1 \end{cases} \Rightarrow a+b+c = 0 + \frac{2}{3} + 1 = \frac{5}{3}$

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} \xrightarrow{x > 0} \begin{cases} x + \frac{1}{x} \in [2, +\infty) \\ 2\sqrt{x + \frac{1}{x}} \in [2\sqrt{2}, +\infty) \end{cases} \Rightarrow R_f = [2 + 2\sqrt{2}, +\infty)$

ب) $y = (3 - \sin x)(1 + \sin x) = -\sin^2 x + 2\sin x + 3$
 $\sin x \rightarrow \begin{cases} \textcircled{1} \Rightarrow y=0 \\ \textcircled{2} \Rightarrow y=4 \\ -\frac{1}{2} = 1 \Rightarrow y=4 \end{cases} \Rightarrow R = [0, 4]$

$f(x) = -\frac{1}{x} x + 10 \quad D_f = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
 $f(1) + f(2) + \dots + f(9) = (9 \times 10) + \left(-\left(\frac{1+2+3+\dots+9}{9}\right)\right) = 90 - \left(\frac{45}{9}\right)$
 $90 - 15 = 75$

$a=0 \rightarrow f(x) = \sqrt{bx+c}$ چون $f(x)$ فقط یک ریشه دارد و دامنه آن $[2, +\infty)$ است داریم
 $f(2)=0 \rightarrow 2b+c=0$
 $f(3)=1 \rightarrow 9b+c=1 \rightarrow b=1, c=-2$
 $g(x) = \sqrt{bx^2+ax-rc} \rightarrow g(x) = \sqrt{x^2+2}$ $R_g = [2, +\infty)$

$ x-r - x-1 = kx$  	$y = kx \xrightarrow{(r, -1)} rk = -1$ $k = -\frac{1}{r}$ (2)	6
الف) $y = x[x]$ $-r < x < -1 \xrightarrow{[x] = -1} y = -rx$ $-1 < x < 0 \xrightarrow{[x] = -1} y = -x$ $0 < x < 1 \xrightarrow{[x] = 0} y = 0$ $1 \leq x < r \xrightarrow{[x] = 1} y = x$ $x = r \xrightarrow{[x] = r} y = r$ $R = [0, r] - \{r\}$	ب) $y = x x - x$ $y = \begin{cases} x^2 - x & x \geq 0 \\ -x^2 - x & x < 0 \end{cases}$  $R = \mathbb{R}$ $R = [-r, r]$	7
$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} = \frac{rx+r}{x^2+x} = \frac{r(x+1)}{x(x+1)} \xrightarrow{x \neq -1} y = \frac{r}{x}$ $R = \mathbb{R} - \{-r, 0\} \Rightarrow a = -r, b = 0$ $a+b = -r+0 = -r$		8
$f(x) = \frac{x - \epsilon\sqrt{x} + r}{x - r\sqrt{x} + 1} = \frac{(\sqrt{x}-1)(\sqrt{x}-r)}{(\sqrt{x}-1)(\sqrt{x}-1)} \xrightarrow{x \neq 1} f(x) = \frac{\sqrt{x}-r}{\sqrt{x}-1}$ $\sqrt{x}=0 \Rightarrow f(x)=r$ $\sqrt{x}=1 \Rightarrow f(x)=1$ $R = (-\infty, 1) \cup (r, +\infty)$ $R = (-\infty, 1) \cup [r, +\infty)$		9
$f(x) = \frac{x^r + rx^r - x - r}{x^r - 1} = \frac{(x^r + rx^r + r)(x-1)}{(x+1)(x-1)} = \frac{(x+1)(x+r)(x-1)}{(x+1)(x-1)}$ $x \neq \pm 1 \Rightarrow f(x) = x+r$ $R = \mathbb{R} - \{f(1), f(-1)\} = \mathbb{R} - \{r, 1\}$ $1+r = r$		10