

14, 5

(الف) $y = (n - 2 \lfloor \frac{n}{2} \rfloor)^2$ $\cdot (2(a - [a]))^2$ ①
 $\downarrow$ متوسّط سیم $\cdot (2(a - [a]))^2 \leq \varepsilon$ ②
 $(2(\frac{n}{2} - \lfloor \frac{n}{2} \rfloor))^2$ $\rightarrow$ صفره بین و صفره $Rf: [0, \varepsilon]$ ✓

(ب) $\lg y = \sin u + \varepsilon$ $3 \leq \lg y \leq 5$ $1 \leq y \leq 10$
 $Rf: [1, 10]$ ✓ $\rightarrow 1 \leq \sin u \leq 1$ $3 \leq \sin u + \varepsilon \leq 5$

$f(n) = \sqrt{\frac{n^2 - \varepsilon}{n^2 - 9}}$ $\Rightarrow \lim_{n \rightarrow \infty} = \frac{\infty}{\infty} = 1$ ② ②
 $Rf: [\frac{2}{3}, \frac{4}{3}]$ ✓ $\rightarrow$ جواب را اول حد $Rf: [\frac{2}{3}, \frac{4}{3}]$ (1, 2, 4, 8)
 $a + b + c = 0 + \frac{2}{3} + 1 = \frac{5}{3}$ ✓

(الف) $f(n) = n + \frac{1}{n} + 2\sqrt{n + \frac{1}{n}} + 1 - 1$ ③
 $f(n) = y = (t+1)^2 - 1 \Rightarrow \sqrt{y+1} = t+1 \rightarrow t = \sqrt{y+1} - 1$
 $y = \sqrt{n + \frac{1}{n}} + 1 - 1$ ✓
 $n + \frac{1}{n} + 1 \geq 0$ $n + \frac{1}{n} \geq -1 \Rightarrow n + \frac{1}{n} \geq -1$ ✓ ④
 $Rf: [1 + 2\sqrt{2}, +\infty)$ ✓

(ب) $y = (3 - \sin u)(1 + \sin u) = -\sin^2 u + 2\sin u + 3$
 $\sin u = \square$ $\begin{cases} \sin u = 1 \rightarrow -1 + 2 + 3 = 4 \\ \sin u = -1 \rightarrow -1 - 2 + 3 = 0 \end{cases}$ $Rf: [0, 4]$ ✓
 $\frac{-b}{2a} = \frac{1}{-2} = -\frac{1}{2}$ ① $\rightarrow \frac{3}{4}$

$$f(n) = -\frac{1}{n}n + 10 \quad 1 \leq n \leq 9 \quad \checkmark$$

① ②

$$f(1) + f(2) + \dots + f(9) = -\frac{1}{n}(n_1 + n_2 + \dots + n_9) + 9(10) = 90 - 10 = 80 \quad \checkmark$$

Σa

$$f(n) = \sqrt{an^2 + bn + c} \longrightarrow Df : (r_0, +\infty) \longrightarrow \text{معر بوجه ٢} \quad \textcircled{5}$$

$$\frac{f}{-f} \quad a=0 \quad \checkmark$$

$$f'(n) = 1 \Rightarrow \begin{cases} 2nb + c = 1 \\ nb + c = 1 \end{cases} \Rightarrow \begin{cases} b = 1 \\ c = -1 \end{cases} \quad \checkmark$$

$$g(n) = \sqrt{bn^2 + an - 1c}$$

$n^2 \quad \quad \quad \varepsilon$

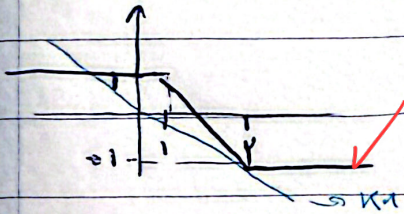
①

$$g(n) = \sqrt{n^2 + \varepsilon} \longrightarrow Rf : (r_0, +\infty)$$

$$\sqrt{\varepsilon} = \varepsilon < 1 \text{ min}$$

$$|u-r| - |u-1| = Ku$$

$$-1 \leq K \leq 1 \quad K = -\frac{1}{r}$$

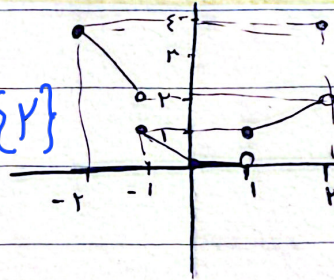


(4)

(5)

الف) $y = u[n]$

$$R_f = [0, r] - \{r\}$$



(6)

(1, 0)

$$-r \leq u < -1 \quad y = ru$$

$$-1 \leq u < 1 \quad y = 0$$

$$-1 \leq u < 0 \quad y = -u$$

✓

$$1 \leq u < r \quad y = u$$

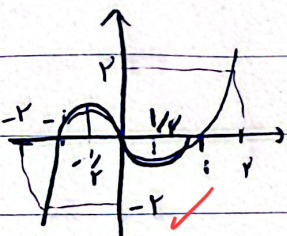
$$u = r \rightarrow y = 2$$

ب) $y = u|u| - u$

$$u^2 - u$$

$$u^2 - u$$

$$u(u-1)$$



$$R_f = [-r, r]$$

$$y = \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n^2+n} \rightarrow \frac{r(n+r)}{n^2+n} = \frac{r(n+1)}{n(n+1)} \xrightarrow{\text{بسط}} \frac{r}{n}$$

(7)

(8)

نمی‌تواند معرور پس y نمی‌تواند معرور شود.

① $u = -1 \quad y = \frac{r}{n} \quad u = -1 \rightarrow y \neq -r$ $\checkmark$ $a+b = \frac{r}{n}$

$$f(u) = \frac{u - \varepsilon\sqrt{u+r}}{u - r\sqrt{u+1}} = \frac{(\sqrt{u-1})(\sqrt{u-r})}{(\sqrt{u-1})^2} \checkmark \rightarrow \sqrt{u} = \frac{r-y}{1-y} \checkmark + \frac{r}{3} + \frac{r}{3} + \frac{r}{3}$$

(9)

(10)

$$R_f = (-\infty, 1) \cup [r, +\infty) \checkmark$$

(11)

(12)

$$f(u) = \frac{u^2 + ru - u - r}{u^2 - r} = \frac{u^2(u+r) - (u+r)}{(u^2-1)} = \frac{(u^2-1)(u+r)}{(u^2-1)} = u+r$$

$$u = 1 \rightarrow y = r$$

$$u = -1 \quad y = 1$$

$$R_f = [r, 1, r]$$

$$1 + r = \sum_{i=1}^{\infty} \frac{1}{2^i}$$

sam