روژین قسوریان تکلیف ۴

Subject:

Year. Month. Date. ( ) Name.

① الف) $y=\left(2\left(\underbrace{\left(\frac{x}{2}\right)-\left[\frac{x}{2}\right]}\right)\right)^2$ $a-[a]$ 

$\longrightarrow 0\le a-[a]<1 \longrightarrow 0\le 2(a-[a])<2 \longrightarrow 0\le (2(a-[a]))^2<4$

$R_f=[0 \quad 4)$

ب) $-\sin x+\log y-4=0$ $\xrightarrow{\sin x=1} \log y=5 \quad y=10^5$

$\xrightarrow{\sin x=-1} \log y=3 \quad y=10^3$

$R_f=[10^3 \quad 10^5]$

② $f(x)=\sqrt{\frac{x^2-4}{x^2-9}}$ (برد عبارت زیر رادیکال) $\xrightarrow{x^2=0} \sqrt{\frac{4}{9}}=\frac{2}{3}$

$\xrightarrow{x^2\to+\infty} \sqrt{1}=1$

$\xrightarrow{\text{خروج صفر را بشود}} (-\infty \quad \frac{2}{3}]\cup(1,+\infty)$

$\xrightarrow{\text{جذر رادیکال}} R=[0 \quad \sqrt{\frac{2}{3}}]\cup(1,+\infty)$

③ $f(x)=x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}}+1-1 \longrightarrow (t^2+2t+1)-1=y$

$(t+1)^2=y+1 \longrightarrow \left(\sqrt{x+\frac{1}{x}}+1\right)^2=y+1$

$2\le x+\frac{1}{x} \longrightarrow \sqrt{2}\le\sqrt{x+\frac{1}{x}} \longrightarrow \sqrt{2}+1\le\sqrt{x+\frac{1}{x}}+1 \longrightarrow$

$\longrightarrow 2+1+2\sqrt{2}\le\left(\sqrt{x+\frac{1}{x}}+1\right)^2 \longrightarrow 2+2\sqrt{2}\le\left(\sqrt{x+\frac{1}{x}}+1\right)^2-1$

$R_f=[2+2\sqrt{2} \ , +\infty)$

ب) $y=(3-\sin x)(1+\sin x) = -\sin^2 x + 2\sin x + 3 = y$

$\xrightarrow{\sin x = 1} y = 4$

$\xrightarrow{\sin x = -1} y = 0$

$\xrightarrow[\frac{2}{2}=1]{-\frac{b}{2a}} y = 4$

$R_f = [0 \quad 4]$

④ $f(x) = -\frac{1}{3}x + 10 \qquad D_f = \{1, 2, 3, 4, \dots 9\}$

$-\frac{1}{3}(1)+10 + -\frac{1}{3}(2)+10 + \dots + -\frac{1}{3}(9)+10 = -\frac{1}{\cancel{3}}\left(\frac{\cancel{9}^{3}\times\cancel{10}^{5}}{\cancel{2}}\right) + 90$

$= -15 + 90 = \boxed{75}$

⑤ $f(x) = \sqrt{ax^2+bx+c} \xrightarrow{[2, +\infty)} \underline{a=0} \qquad bx+c$

$\longrightarrow 2b+c=0$

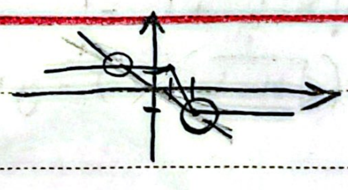
$\xrightarrow{(3\ 1)} 3b+c=1$

$\underline{b=1} \qquad \underline{c=-2}$

$g(x) = \sqrt{x^2-4} \longrightarrow R_g = [0, +\infty)$

⑥ $|x-2|-|x-1| = kx$

$(2\ -1), (0, 0)$



$-\frac{1}{2} \Rightarrow k = -\frac{1}{2}$

⑦ $y = x[x]$

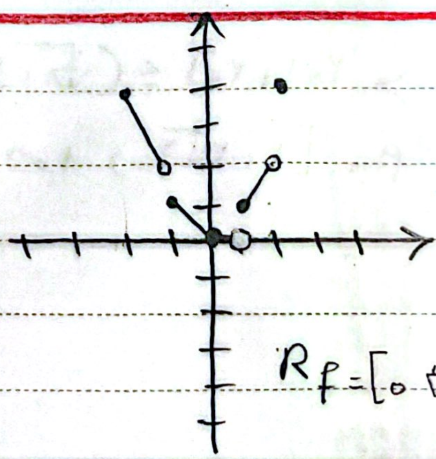
$-2 \le x < -1 \longrightarrow -2x$

$-1 \le x < 0 \longrightarrow -x$

$0 \le x < 1 \longrightarrow 0$

$1 \le x < 2 \longrightarrow x$

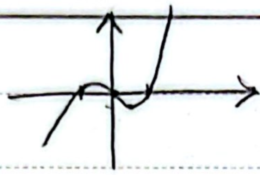
$x = 2 \longrightarrow 2x$



$R_f = [0 \ 4] - \{2\}$

$x(x-1$

ب) $y = x|x| - x$ $\xrightarrow{0 \le x} x^۲ - x$, $\frac{۱}{۲} - \frac{۱}{۲}$; $\xrightarrow{x \le 0} -x^۲ - x$, $-x(x+1)$



$R_f = \mathbb{R}$

۸) $y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^۲+x} \xrightarrow{x \neq 0,-1} y = \frac{x+1+x+1}{x^۲+x}$

$y = \frac{۲x+۲}{x^۲+x}$ $\quad y = \frac{۲(x+1)}{x(x+1)} \xrightarrow{x \neq 0,-1} y = \frac{۲}{x}$ $\xrightarrow{x \neq 0} y \neq 0$, $\xrightarrow{x \neq -1} y \neq -۲$

$a + b = (-۲ + 0) = -۲$

۹) $f(x) = \frac{x - ۴\sqrt{x} + ۳}{x - ۲\sqrt{x} + 1} = \frac{(\sqrt{x}-۳)(\sqrt{x}-1)}{(\sqrt{x}-1)^۲} \xrightarrow{x \neq 1} y = \frac{\sqrt{x}-۳}{\sqrt{x}-1}$

$x = 0 \to y = ۳$
$x = +\infty \to y \simeq 1$

$\Big\}$ $\xrightarrow[\text{مرز}]{\text{خرج صفر}}$ $R = (-\infty, 1) \cup [۳, +\infty)$

۱۰) $f(x) = \frac{x^۳ + ۲x^۲ - x - ۲}{x^۲ - 1} = \frac{x^۲(x+۲) - (x+۲)}{(x-1)(x+1)}$

$= \frac{(x+۲)(x^۲-1)}{(x^۲-1)} \xrightarrow{x \neq \pm 1} y = x + ۲$ $\xrightarrow{x \neq +1} y \neq ۳$, $\xrightarrow{x \neq -1} y \neq 1$ $\longrightarrow 1 + ۳ = ۴$