

$$y = (x - 2[\frac{x}{4}])^2 = (2(\frac{x}{2}) - [\frac{x}{2}])^2 = 4(\frac{x}{2} - [\frac{x}{2}])^2 \rightarrow R_p = [0, 4] \quad \checkmark$$

$$R = [0, 1)$$

ب) $-\sin x + \log y - 4 = 0 \rightarrow \log y - 4 = \sin x \rightarrow -1 \leq \log y - 4 \leq 1 \rightarrow 3 \leq \log y \leq 5$

$$\left. \begin{array}{l} \log y \geq 3 \rightarrow y \geq 10^3 \\ \log y \leq 5 \rightarrow y \leq 10^5 \end{array} \right\} 10^3 \leq y \leq 10^5 \rightarrow R_p = [10^3, 10^5] \quad \checkmark$$

$$y = \sqrt{\frac{x^2 - 4}{x^2 - 9}} \rightarrow y^2 = \frac{x^2 - 4}{x^2 - 9} \rightarrow x^2 = -\frac{4y^2 + 4}{y^2 - 1} \rightarrow x^2 = \frac{4y^2 - 4}{y^2 - 1} \rightarrow x = \pm \sqrt{\frac{4y^2 - 4}{y^2 - 1}}$$

$$\rightarrow \frac{4y^2 - 4}{y^2 - 1} \geq 0 \rightarrow \frac{-1}{x^2 - 9} + \frac{y^2}{x^2 - 9} \rightarrow R_p = [0, \frac{y}{x}] \cup (1, +\infty) = [a, b] \cup (c, +\infty)$$

نقطه از ۰ می باشد

$$\rightarrow \begin{cases} a = 0 \\ b = \frac{y}{x} \\ c = 1 \end{cases} \left\{ \begin{array}{l} a + b + c = 0 + \frac{y}{x} + 1 = \frac{y}{x} + 1 \end{array} \right. \quad \checkmark$$

الف) $y = x + \frac{1}{x} + \sqrt{x + \frac{1}{x}} \rightarrow x + \frac{1}{x} \rightarrow [2, +\infty)$

$$\rightarrow y = x + \frac{1}{x} + \sqrt{x + \frac{1}{x}} \rightarrow R_p = (2 + \sqrt{2}, +\infty) \quad R_f = [2 + \sqrt{2}, +\infty)$$

ب) $y = (3 - \sin x)(1 + \sin x) = 3 + 3\sin x - \sin x - \sin^2 x \rightarrow y = -\sin^2 x + 2\sin x + 3$

$$\left\{ \begin{array}{l} \min \Rightarrow \sin x = -1 \Rightarrow y = -(-1)^2 + 2(-1) + 3 = -1 + (-2) + 3 = 0 \\ \max \Rightarrow \sin x = 1 \Rightarrow y = -(1)^2 + 2(1) + 3 = -1 + 2 + 3 = 4 \end{array} \right\} \Rightarrow R_p = [0, 4] \quad \checkmark$$

$f(x) = \frac{1}{x^2}x + 10, 1 \leq x \leq 9 \rightarrow f(1) + f(2) + f(3) + \dots + f(9) = ?$

$$\rightarrow \left(\frac{1}{1^2}(1) + 10 \right) + \left(\frac{1}{2^2}(2) + 10 \right) + \dots + \left(\frac{1}{9^2}(9) + 10 \right) = \frac{1}{9} (1 + 2 + \dots + 9) + (9 \times 10) = \frac{-45}{9} + 90 = -5 + 90 = 85$$

$$= \frac{-45 + 810}{9} = \frac{765}{9} = 85 \quad \checkmark$$

$f(x) = \sqrt{ax^2 + bx + c}, D_f = [2, +\infty) \rightarrow f(x) = \sqrt{(x-2)^2} \rightarrow ax^2 + bx + c = (x-2)^2$

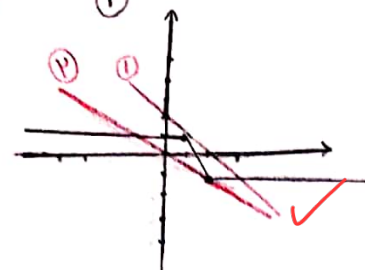
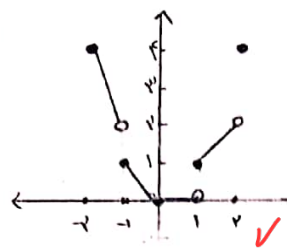
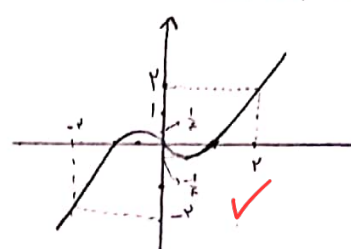
با توجه به اینکه دامنه $x \geq 2$ است، عبارت ما یک تابع خطی است.

$$\rightarrow ax^2 + bx + c = x^2 + 4x + 4 \rightarrow \begin{cases} a = 1 \\ b = 4 \\ c = 4 \end{cases}$$

$$\rightarrow x^2 + x - 1 \rightarrow \text{ext} \left\{ \begin{array}{l} \frac{-b}{2a} = \frac{-1}{2} \\ f(\frac{-1}{2}) = \frac{1}{4} - \frac{1}{4} - 1 = -1 \end{array} \right\} R = [-1, +\infty) \rightarrow R_p = [0, +\infty)$$

$f(2) = 0, f(3) = 1 \rightarrow \begin{cases} b = 1 \\ c = -2 \end{cases}$

$g(x) = \sqrt{x^2 + 4} \quad R = [2, +\infty)$

$ x-2 - x-1 = kx$ 	طبق شکل فقط $y=kx$ می‌تواند ۲ حالت داشته باشد: نمودار را در نقاط قطع کرد ۱- از آنبابی که $y=kx$ عوض از میداندار حالت ① قبل قبول نیست و فقط ② که از مداری کز در قایل قبول است. ← فقط $y=kx$ از نقاط ۱ و ۲ کز کرده است $y=kx = \frac{-1}{2}x \Rightarrow \boxed{k = \frac{-1}{2}}$ ②
الف) $y = x[x]$ $x < -1 \xrightarrow{[x]=-1} y = -x$ $-1 < x < 0 \xrightarrow{[x]=0} y = 0$ $0 < x < 1 \xrightarrow{[x]=0} y = 0$ $1 < x < 2 \xrightarrow{[x]=1} y = x$ $x = 2 \xrightarrow{[x]=2} y = 4$  $\Rightarrow R_f = [0, 4] - \{1\}$	ب) $y = x x - x$ $x \geq 0 \rightarrow x^2 - x = x(x-1)$ $x < 0 \rightarrow -x^2 - x = -x(x+1)$  $\Rightarrow R_f = [-2, 2]$ ②
$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} = \frac{(x+1)+x+1}{x^2+x} = \frac{2x+2}{x^2+x} = \frac{2(x+1)}{x(x+1)} \xrightarrow{x \neq -1} y = \frac{2}{x}$ $\rightarrow x = \frac{y}{2}$ $\begin{cases} y \neq 0 \\ x \neq -1 \rightarrow y \neq -2 \end{cases} \Rightarrow R_f = \mathbb{R} - \{0, -2\}$ $\mathbb{R} - \{a, b\} \xrightarrow{a=0, b=-2} \begin{cases} a+b = 0+(-2) = -2 \end{cases}$ ②	②
$f(x) = \frac{x - \sqrt{x} + 3}{x - 2\sqrt{x} + 1} = \frac{(\sqrt{x}-3)(\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}-1)} \xrightarrow{x \neq 1} y = \frac{\sqrt{x}-3}{\sqrt{x}-1} \rightarrow \sqrt{x} = \frac{-y+3}{y-1}$ $\rightarrow \sqrt{x} = \frac{y-3}{y-1} \rightarrow \frac{1}{10p-1} + \frac{y}{1} \rightarrow R_f = (-\infty, 1) \cup [3, +\infty)$ ②	②
$f(x) = \frac{x^3 + x^2 - x - 2}{x^2 - 1} = \frac{(x-1)(x^2 + 3x + 2)}{(x-1)(x+1)} = \frac{(x-1)(x+1)(x+2)}{(x-1)(x+1)} \xrightarrow{x \neq -1} y = x+2$ $\rightarrow R_f = \mathbb{R} - \{3, 1\} \rightarrow y+1 = [1, 3]$ $\int x=1 \text{ جای } y = x+2 = 1+2 = 3$ $\int x=-1 \text{ جای } y = x+2 = -1+2 = 1$ ②	②