تکلیف ٤ :   به نام خدا

A نیکا دهقانی / دوازدهم / دهتر

۱۹٫۵

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۱) الف) $\left(x-2\left[\frac{x}{2}\right]\right)^2 \xrightarrow{R_f} \left(2\left(\frac{x}{2}-\left[\frac{x}{2}\right]\right)\right)^2$ $\quad \bigcirc - [\bigcirc] \in [0,1)$

$0 \leq \frac{x}{2}-\left[\frac{x}{2}\right] < 1 \xrightarrow{\times 2} 0 \leq 2\left(\frac{x}{2}-\left[\frac{x}{2}\right]\right) < 2 \xrightarrow{\text{به توان ۲}} 0 \leq y < 4$

$\rightarrow R_f = [0, 4) \rightarrow 2.$

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ب) $-\sin x + \log y - 4 = 0 \xrightarrow{R_f} \log y = \sin x + 4$

$\xrightarrow{-1 \leq \sin x \leq 1} 3 \leq \sin x + 4 \leq 5 \rightarrow 3 \leq \log y \leq 5$

(I) $10^3 \leq y \leq 10^5$ و (II) $y > 0 \xrightarrow{\cap} R_f = [10^3, 10^5] \rightarrow 2.$

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۲) $f(x) = \sqrt{\frac{x^2-4}{x^2-9}} \xrightarrow{R_f} [a, b] \cup (c, +\infty)$, $a+b+c = ?$

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$y = \sqrt{\frac{x^2-4}{x^2-9}} \rightarrow y^2 = \frac{x^2-4}{x^2-9} \rightarrow y^2x^2 - 9y^2 = x^2 - 4$

$\rightarrow y^2x^2 - x^2 = 9y^2 - 4 \rightarrow x^2(y^2-1) = 9y^2-4 \Rightarrow x^2 = \frac{9y^2-4}{y^2-1}$

$\Rightarrow x = \sqrt{\frac{9y^2-4}{y^2-1}}$ زیر رادیکال ≥ 0 (I) $\frac{9y^2-4}{y^2-1} \geq 0 \xrightarrow{\text{تعیین علامت}}$ $-1 \quad -\frac{2}{3} \quad \frac{2}{3} \quad 1$: $+ \; - \; + \; - \; +$

(II) $y \geq 0 \xrightarrow{(I) \cap (II)} \left[0, \frac{2}{3}\right] \cup (1, +\infty)$ (a, b, c)

$\Rightarrow a+b+c = \frac{5}{3} \rightarrow 2.$

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۳) الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x+\frac{1}{x}} \xrightarrow{R_f} y = x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}}+1-1$

$\rightarrow y = \left(\sqrt{x+\frac{1}{x}}+1\right)^2 - 1$ $\quad x+\frac{1}{x} \in (-\infty, -2] \cup [2, +\infty)$ زیر رادیکال ≥ 0 $\rightarrow x+\frac{1}{x} \in [2, +\infty)$ می‌باشد

$\rightarrow \sqrt{2} \leq \sqrt{x+\frac{1}{x}} \xrightarrow{+1} \sqrt{2}+1 \leq \sqrt{x+\frac{1}{x}}+1 \xrightarrow{\text{به توان ۲}} 3+2\sqrt{2} \leq \left(\sqrt{x+\frac{1}{x}}+1\right)^2$

$\xrightarrow{-1} 2+2\sqrt{2} \leq \left(\sqrt{x+\frac{1}{x}}+1\right)^2 - 1 \Rightarrow R_f = [2+2\sqrt{2}, +\infty)$ 2.

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$\rightarrow \sin x \in [-1,1]$

ب) $y = (3 - \sin x)(1 + \sin x) \xrightarrow{Rf} y = -\sin^2 x + 2\sin x + 3$

$\rightarrow \min \sin x \rightarrow \sin x = -1 \rightarrow y = 0$

$\rightarrow \max \sin x \rightarrow \sin x = 1 \rightarrow y = 4$

$\rightarrow \frac{-b}{2a} = \frac{-2}{-2} = 1$ ✓ $\rightarrow \sin x = 1 \rightarrow y = 4$

$\Rightarrow Rf = [0, 4]$

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۴) (۲)

$f(x) = -\frac{1}{2}x + 10 \longrightarrow Df = \mathbb{N}$ دو رقمی ، جمع $Rf = ?$

$-\frac{1}{2}x_1 + 10 - \frac{1}{2}x_2 + 10 - \frac{1}{2}x_3 + 10 + \cdots - \frac{1}{2}x_9 + 10 =$

$-\frac{1}{2}(x_1 + x_2 + \cdots + x_9) + (10 \times 9) \Rightarrow -\frac{1}{2}\left(\frac{(1+9)\times 9}{2}\right) + 90$

$-\frac{1}{2}(45) + 90 = -15 + 90 = 75 \rightarrow$ ۹.

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۵)

$f(x) = \sqrt{ax^2 + bx + c} \rightarrow Df = [2, +\infty)$, $f(3) = 1$

$g(x) = \sqrt{bx^2 + ax - 2c}$

$0 \leq$ زیر رادیکال $\rightarrow ax^2 + bx + c \geq 0 \rightarrow x \geq 2 \rightarrow$ باید معادله درجه یک باشد.

$\rightarrow a = 0 \Rightarrow bx + c \geq 0 \rightarrow x \geq -\frac{c}{b} \rightarrow -\frac{c}{b} = 2$, $c = -2b$

$f(3) = \sqrt{b(3) - 2b} = \sqrt{b} = 1 \rightarrow b = 1$, $c = -2$

$g(x) = \sqrt{x^2 + 4} \Rightarrow y^2 = x^2 + 4 \rightarrow y^2 - 4 = x^2 \rightarrow x = \pm\sqrt{y^2 - 4}$

زیر رادیکال $\rightarrow y^2 \geq 4 \rightarrow$ (I) $y \geq 2 \cup y \leq -2$ و (II) $y \geq 0$

(I) $\cap$ (II) $= [2, +\infty) = Rf \rightarrow$ ۲.

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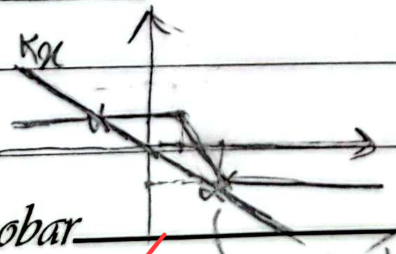
۶)

$|x-2| - |x-1| = kx \rightarrow$ ۲ جواب

شکل



$\rightarrow y = kx \xrightarrow{(2,-1)} -1 = 2k \rightarrow k = -\frac{1}{2}$

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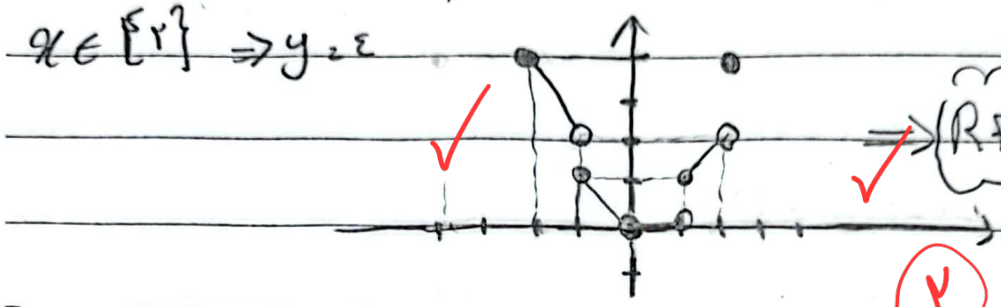
(۲)

باید فقط نقطه را قطع کند!

الف) $y = x[x]$ $[-2,2]$ ← $x \in [-2,-1) \Rightarrow y = -2x$ (۷)

$x \in [-1,0) \Rightarrow y = -x$ / $x \in [0,1) \Rightarrow y = 0$ / $x \in [1,2) \Rightarrow y = x$

$x \in [2] \Rightarrow y = 4$



$\Rightarrow R_f = [0,4] - \{2\}$

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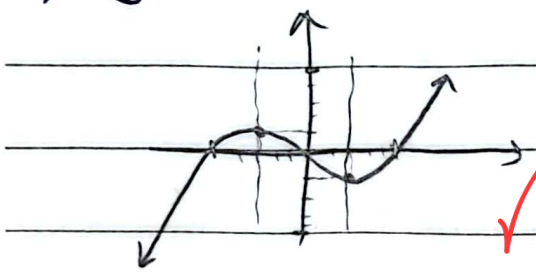
ب) $y = x|x| - x$ $[-2,2]$ ←



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$-x^2 - x$ | $x^2 - x$

$x = 2 \to y = 2$

$x = -2 \to y = -2$

$-\frac{b}{2a} = -\frac{1}{2}$ ↓ $y = \frac{1}{4}$

$-\frac{b}{2a} = \frac{1}{2}$ ↓ $y = -\frac{1}{4}$

$\Rightarrow R_f = [-2,2]$

$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} \to R_f = R - \{a,b\} \to a+b = ?$ (۸)

$\Rightarrow y = \frac{x + x+1 + 1}{x^2+x} = \frac{2x+2}{x^2+x} = \frac{2(x+1)}{x(x+1)}$ $x \neq -1$ $\to \frac{2}{x} = y$

اگر $x = -1 \to \frac{2}{-1} = -2$ و مخرج کسر $= \frac{0}{x} = 0 \to a = -2, b = 0$ برقرار

$\to a + b = -2 + 0 = -2$ ؟

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$f(x) = \frac{x - 4\sqrt{x} + 4}{x - 2\sqrt{x} + 1}$ $R_f \to y = \frac{(\sqrt{x}-2)(\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}-1)}$ اگر $x \neq 1$ (۹)

$y = \frac{\sqrt{x}-2}{\sqrt{x}-1} \Rightarrow y\sqrt{x} - y = \sqrt{x} - 2$

$\Rightarrow y\sqrt{x} - \sqrt{x} = y - 2 \to \sqrt{x} = \frac{y-2}{y-1} \Rightarrow \frac{y-2}{y-1} \geq 0$

$\Rightarrow R_f = (-\infty, 1] \cup [2, +\infty)$

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$R_f = (-\infty, 1) \cup [2, +\infty)$

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$f(x) = \frac{x^3+2x^2-x-2}{x^2-1}$ $Rf \rightarrow$

$x^3+2x^2-x-2 \mid x^2-1$

$-x^3+x$

$2x^2-2$

$x+2$ (2)

$\Rightarrow y = \frac{(x^2-1)(x+2)}{(x^2-1)}$ $x \neq \pm 1$ $\rightarrow y = x+2$

$\rightarrow x=1 \rightarrow y=3$

$\rightarrow x=-1 \rightarrow y=+1$

$Rf = \mathbb{R} - \{3, +1\} \Rightarrow +3+1 = (4) \rightarrow 2.$ (1)