الف) $y = \left(\underbrace{x - 2\left[\frac{x}{2}\right]}\right)^2$

$\downarrow$

$\left(2\left(\underbrace{\frac{x}{2} - \left[\frac{x}{2}\right]}_{[0,1)}\right)\right)^2 \longrightarrow \left([0,2)\right)^2 \longrightarrow R_f = [0,4)$

ب) $-\sin x + \log y - 4 = 0 \longrightarrow \log y = \sin x + 4$

$-1 \leq \sin x \leq 1 \longrightarrow 3 \leq \sin x + 4 \leq 5 \longrightarrow 3 \leq \log y \leq 5$

$\rightarrow 10^3 \leq y \leq 10^5$

$f(x) = \sqrt{\frac{x^2-4}{x^2-9}} \xrightarrow{\text{توان ۲}} y^2 = \frac{x^2-4}{x^2-9} \longrightarrow y^2x^2 - 9y^2 = x^2 - 4$ ۳

$y^2x^2 - x^2 = 9y^2 - 4$

$(3y-2)(3y+2)$

$\frac{9y^2-4}{y^2-1} \geq 0$

$(y-1)(y+1)$

$x = \pm\sqrt{\frac{9y^2-4}{y^2-1}} \leftarrow x^2 = \frac{9y^2-4}{y^2-1} \leftarrow x^2(y^2-1) = 9y^2-4$

y		-1		$-\frac{2}{3}$		$\frac{2}{3}$		1	
	$+$	ت	$-$	0	$+$	0	$-$	ت	$+$

$\longrightarrow R_f = \left[0, \frac{2}{3}\right] \cup (1, +\infty)$

$a + b + c = 0 + \frac{2}{3} + 1 = \frac{5}{3}$

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}}$ $\rightarrow$ عبارت زیر رادیکال $\rightarrow [2, +\infty)$

$a + \frac{1}{a} \begin{cases} a > 0 \rightarrow [2, +\infty) \\ a < 0 \rightarrow (-\infty, -2] \end{cases}$

$f(x) = \left(\sqrt{x + \frac{1}{x}} + 1\right)^2 - 1$

$\sqrt{x + \frac{1}{x}} = \sqrt{2} \longrightarrow (\sqrt{2} + 1)^2 - 1$

$2 + 1 + 2\sqrt{2} - 1 = 2 + 2\sqrt{2}$ ۳

$\sqrt{x + \frac{1}{x}} = \infty \longrightarrow y = \infty$

$\Rightarrow R_f = [2 + 2\sqrt{2}, +\infty)$

ب) $y = (3 - \sin x)(1 + \sin x) = 3 + 3\sin x - \sin x - \sin^2 x =$

$\leftarrow -(\sin^2 x - 2\sin x - 3) \leftarrow -\sin^2 x + 2\sin x + 3$

$-\left((\sin x - 1)^2 - 4\right) \longrightarrow -1 \leq \sin x \leq 1 \longrightarrow -2 \leq \sin x - 1 \leq 0 \longrightarrow$

$0 \leq -\left((\sin x - 1)^2 - 4\right) \leq 4 \leftarrow -4 \leq (\sin x - 1)^2 - 4 \leq 0 \leftarrow 0 \leq (\sin x - 1)^2 \leq 4$

$R_f = [0, 4]$

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$f(x) = -\frac{1}{3}x + 10$ $\qquad D_f = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$-\frac{1}{3} + \frac{30}{3} = \frac{29}{3}$ $\qquad -\frac{9}{3} + \frac{30}{3} = \frac{21}{3}$

$-\frac{2}{3} + \frac{30}{3} = \frac{28}{3}$

$-\frac{3}{3} + \frac{30}{3} = \frac{27}{3}$

$\longrightarrow R_f = \frac{29 + 28 + 27 + \dots + 21}{3} = 75$

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$R_g = ?$ $\qquad f(3) = 1$ $\qquad f(x) = \sqrt{ax^2 + bx + c}$

$g(x) = \sqrt{bx^2 + ax - 2c}$ $\qquad D_f = [2, +\infty)$

$ax^2 + bx + c \geq 0$

$\begin{array}{c|c} & 2 \\ \hline - & + \end{array}$ $\qquad a = 0$

$\sqrt{x^2 + 4} = \sqrt{[4, +\infty)} \to [2, +\infty)$

$R_g = [2, +\infty)$

$f(2) = 0 \to \begin{cases} 2b + c = 0 \\ 3b + c = 1 \end{cases}$

$f(3) = 1$

$b = 1$ $\qquad c = -2$

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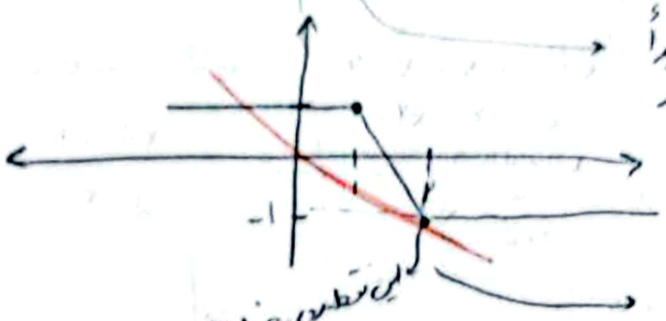
$|x-2| - |x-1| = kx$ $\qquad \to 2 \to -1$

۲ جواب دارد $\qquad 1 \to 1$



باید از مبدأ عبور کند

شیب خط با داشتن ۲ نقطه مشخص است $\longrightarrow k < 0$

این تقاطع جواب است

$2k = -1$

$k = -\frac{1}{2}$

$-\frac{1}{2}x$

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الف) $y = x[x]$

$[-2, -1) \to -2x \to |^{-2}_{4} \quad |^{-1}_{2}$

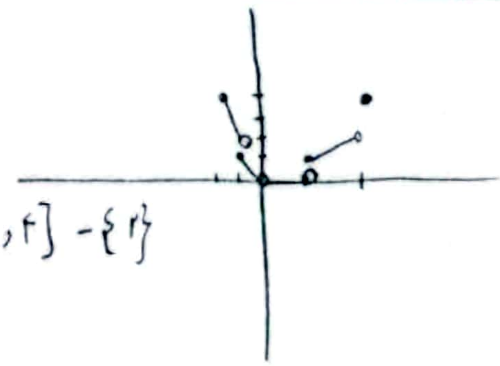
$[-1, 0) \to -x \to |^{-1}_{1} \quad |^{0}_{0}$

$[0, 1) \to 0 \to$

$[1, 2) \to x \to |^{1}_{1} \quad |^{2}_{2}$

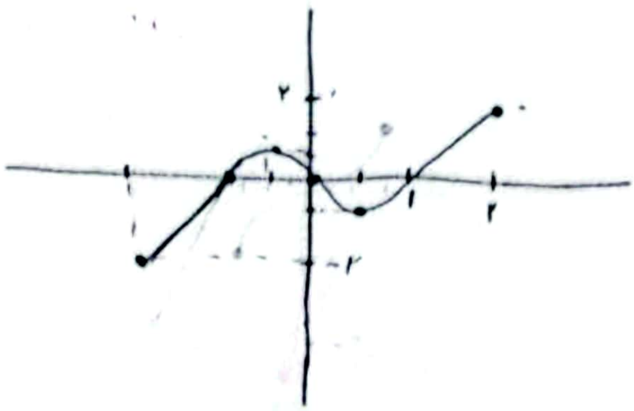
$\{2\} \to 4$

$R_f = [0, 4] - \{2\}$



ب) $y = x|x| - x \begin{cases} x^2 - x & x \geq 0 \\ -x^2 - x & x < 0 \end{cases}$

$\begin{array}{c\|c} \cdot & \cdot \end{array}$	$\begin{array}{c\|c} 2 & 2 \end{array}$	$\begin{array}{c\|c} S & \begin{matrix} \frac{1}{2} \\ -\frac{1}{4} \end{matrix} \end{array}$
$\begin{array}{c\|c} \cdot & \cdot \end{array}$	$\begin{array}{c\|c} -2 & -2 \end{array}$	$\begin{array}{c\|c} S & \begin{matrix} -\frac{1}{2} \\ \frac{1}{4} \end{matrix} \end{array}$



$R_f = [-2, 2]$

$$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{\underset{x(x+1)}{x^2 + x}}$$

$$y = \frac{x+1+x+1}{x^2+x} = \frac{2x+2}{x^2+x} \rightarrow \frac{2\cancel{(x+1)}}{x\cancel{(x+1)}} = \frac{2}{x}$$

$a + b = ?$ $\quad \mathbb{R} - \{a, b\}$

صفر در برد نیست -

$\frac{2}{x} \xrightarrow{-1} -2$ هم در برد نیست چون ۱- در دامنه نیست

$\mathbb{R} - \{0, -2\}$

$a + b = -2$

$$f(x) = \frac{x - 4\sqrt{x} + 3}{x - 2\sqrt{x} + 1} = \frac{\cancel{(\sqrt{x} - 1)}(\sqrt{x} - 3)}{(\sqrt{x} - 1)^2} = \frac{\sqrt{x} - 3}{\sqrt{x} - 1}$$

یک در دامنه نیست (در نظر بگیر)

$$y = \frac{\sqrt{x} - 3}{\sqrt{x} - 1} \rightarrow y\sqrt{x} - y = \sqrt{x} - 3$$

$$y\sqrt{x} - \sqrt{x} = y - 3$$

$$\sqrt{x}(y-1) = y - 3 \rightarrow \overset{x \geq 0}{\sqrt{x}} = \frac{y-3}{y-1} \rightarrow \frac{y-3}{y-1} \geq 0$$

$R_f = (-\infty, 1) \cup [3, +\infty)$

$\begin{array}{ccccc} & 1 & & 3 & \\ + & \circ & - & \bullet & + \end{array}$

$$f(x) = \frac{x^3 + 2x^2 - x - 2}{x^2 - 1} = \frac{(x-2)\cancel{(x+1)}\cancel{(x+1)}}{\cancel{(x-1)}\cancel{(x+1)}} = x + 2$$

[illegible]

$$\begin{array}{l\|l} x^3 + 2x^2 - x - 2 & x - 1 \\ \hline x^3 - x^2 & x^2 + 3x + 2 \\ \hline 3x^2 - x - 2 & \downarrow \\ 3x^2 - 3x & (x+1)(x+2) \\ \hline 2x - 2 & \end{array}$$

یک، منفی یک در دامنه نیست

$-1 \rightarrow 1$

$1 \rightarrow 3$

$R_f = \mathbb{R} - \{1, 3\}$

$1 + 3 = 4$