

$$f(x+1) + f(x+2) = 2x + f(x)$$

$$f(x) = ax + b \text{ (خطی)}$$

$$\hookrightarrow a(x+1) + b + a(x+2) + b = 2x + a + b$$

$$2ax + 3a + 2b = 2x + a + b \Rightarrow x=0 \Rightarrow 3a + 2b = a + b \Rightarrow b=0$$

$$\hookrightarrow 2ax + 2b = 2x + b \Rightarrow 2a = 2 \Rightarrow a=1 \quad x=1 \Rightarrow 3a + 5b = 2 + a + b \Rightarrow 2a = 2 \Rightarrow a=1$$

$$\Rightarrow f(x) = x$$

$$y = \sqrt{f(x)} \Rightarrow y = \sqrt{-x^2 + x} \rightarrow R = \sqrt{(-\infty, y_{\max})}$$

$$x_{\max} = \frac{-b}{2a} = \frac{-1}{-2} = \frac{1}{2}$$

$$y_{\max} = \sqrt{-\frac{1}{4} + \frac{1}{4}} = \frac{1}{2} \rightarrow R = \sqrt{(-\infty, \frac{1}{2}]} = [\frac{0}{2}, \frac{1}{2}]$$

$$f(x) = \frac{x^2 - 2x}{x+2}$$

$$\hookrightarrow \frac{a+b}{c} = \frac{-2+2}{(-1)2} = -1 \Rightarrow f(-1) = \frac{(-1)^2 - 2(-1)}{-1+2} = \frac{-1+2}{1} = 1$$

$$\Rightarrow y = \frac{x(x-2)}{x+2} = x^2 - 2x$$

$$R_f[y_{\min}, +\infty) \Rightarrow x_{\min} = \frac{-b}{2a} = \frac{-2}{2} = -1$$

$$y_{\min} = 1 - 2 = -1$$

$$x \neq -2 \Rightarrow y \neq 1$$

$$\Rightarrow R_f = [-1, +\infty) - \{1\}$$

$$y = \sqrt{x^2 + ax^2 + bx - 1}$$

$$\xrightarrow{\text{مربع ضعیف}} x^2 + ax^2 + bx - 1 = (x-2)^2 = x^2 - 4x + 4 - 1 = x^2 - 4x + 3$$

$$\Rightarrow y = \sqrt{(x-2)^2} = x-2$$

$$\hookrightarrow -4 \leq x \leq 12 \rightarrow -1 \leq x-2 \leq 10$$

$$\Rightarrow D_f = [-4, 12]$$

$$R_f = [-1, 10]$$

$$y_1 = \left(\frac{x+1}{1-x}\right)^{\frac{1}{2}} = \sqrt{\frac{x+1}{1-x}}$$

$$\Rightarrow 1-x \neq 0 \Rightarrow x \neq 1$$

$$D_1 = \mathbb{R} - \{1\}$$

$$y_2 = \frac{ax^2-1}{x^2+b} \Rightarrow yx^2 + yb = ax^2 - 1$$

$$(y-a)x^2 = -(yb+1) \Rightarrow x^2 = \frac{yb+1}{-y+a} \Rightarrow \frac{b}{-1} = \frac{1}{a} \Rightarrow ab = -1$$

$$y = \mathbb{R} - \{1\} \Rightarrow y \neq a \Rightarrow a=1$$

$$a=1, b=1, a+b=2$$

$$(x^2 = \frac{-y+1}{y+1} = 1 \checkmark)$$

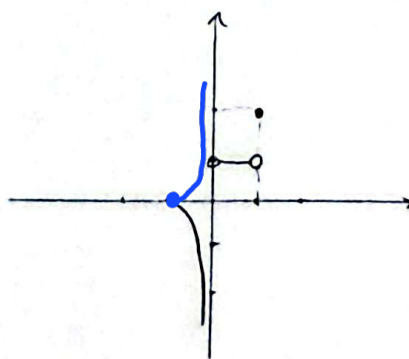
$$\rightarrow a=1, b=-1, a+b=1-1=0$$

$$y = \frac{[x]}{x} + \frac{|x|}{x} \quad (x \neq 0)$$

$$-1 \leq x < 0 \Rightarrow [x] = -1, |x| = -x \Rightarrow \frac{[x]}{x} = \frac{-1}{x} \rightarrow y = \frac{-1}{x} - 1$$

$$0 < x < 1 \Rightarrow [x] = 0, |x| = x \Rightarrow \frac{[x]}{x} = \frac{0}{x} = 0 \rightarrow y = 1$$

$$x=1 \Rightarrow [x] = 1, |x| = 1 \Rightarrow \frac{[x]}{x} = \frac{1}{1} = 1 \rightarrow y = \frac{1}{1} + 1 = 2$$



$$y = [x] + \left[x + \frac{1}{r}\right] \quad D_f: \left[\frac{1}{r}, r\right)$$

$$\frac{1}{r} \leq x < 1 \rightarrow 1 \leq x + \frac{1}{r} < \frac{r}{r} \rightarrow [x] = 0$$

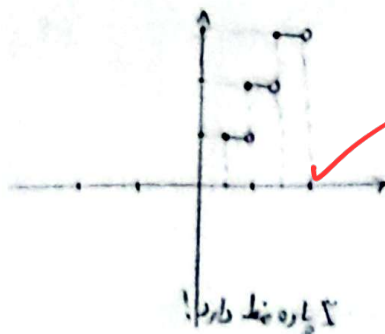
$$\left[x + \frac{1}{r}\right] = 1 \rightarrow y = 1$$

$$1 \leq x < \frac{r}{r} \rightarrow \frac{r}{r} \leq x + \frac{1}{r} < r \rightarrow [x] = 1$$

$$\left[x + \frac{1}{r}\right] = 1 \rightarrow y = r$$

$$\frac{r}{r} \leq x < r \rightarrow r \leq x + \frac{1}{r} < \frac{r}{r} \rightarrow [x] = 1$$

$$\left[x + \frac{1}{r}\right] = r \rightarrow y = r$$



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$$y = \left| \frac{x+1}{ax+r} \right| \xrightarrow{y < 1} \left| \frac{x+1}{ax+r} \right| < 1 \rightarrow \frac{|x+1|^r}{(ax+r)^r} < 1 \rightarrow (x+1)^r < (ax+r)^r \rightarrow x^r + rx + 1 < a^r x^r + 9ax + 9$$

$$(b, +\infty) \rightarrow a^r = 1 \rightarrow a = 1 \rightarrow (x+1)^r < (x+r)^r \rightarrow x^r + rx + 1 < x^r + 9x + 9 \rightarrow -r < x$$

$$D = (-r, +\infty) \checkmark$$

$$\rightarrow a = -1 \rightarrow (x+1)^r < (-x+r)^r \rightarrow x^r + rx + 1 < x^r - 9x + 9 \rightarrow x < 1$$

$$D = (-\infty, 1) \times$$

$$\rightarrow a = 1, b = -r \rightarrow a - b = 1 - (-r) = r \checkmark$$

2

$$|x-1| + |y-1| = r$$

$$x = \frac{r}{2}, y = 1$$

$$\xrightarrow{\text{center, radius}} r(x=1) \rightarrow x=1, y=1$$

$$x < r, y > 1 \rightarrow x - r + y - 1 = r \rightarrow y = -x + r$$

$$\rightarrow rx = r \rightarrow x = r, y = r$$

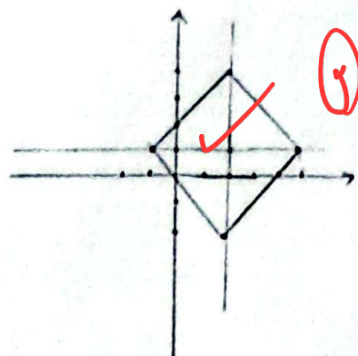
$$x < r, y > 1 \rightarrow r - x + y - 1 = r \rightarrow y = x + r$$

$$\rightarrow rx = -r \rightarrow x = -1, y = 1$$

$$x < r, y < 1 \rightarrow r - x + 1 - y = r \rightarrow y = -x$$

$$\rightarrow rx = r \rightarrow x = r, y = -r$$

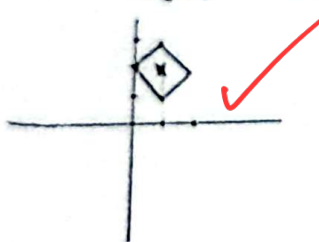
$$x > r, y < 1 \rightarrow x - r + 1 - y = r \rightarrow y = x - r$$



2

$$a) |x-1| + |y-1| = 1 \rightarrow \text{diamond centered at (1,1) with radius 1}$$

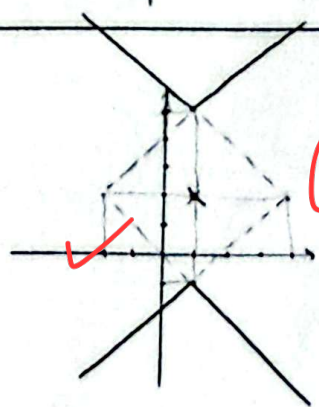
$$x=1, y=2$$



$$b) |x-1| - |y-1| = -r$$

$$|y-1| - |x-1| = r$$

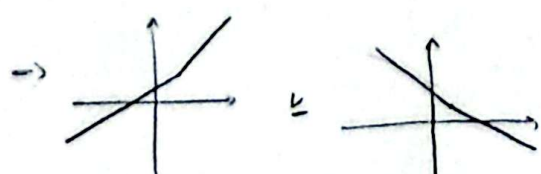
$$y=2, x=1$$



2

$$y = rx + |ax+b| \rightarrow y = (r+a)x + b$$

$$\rightarrow y = (r-a)x - b$$



$$r+a > 0$$

$$a > -r$$

$$r-a > 0$$

$$a < r$$

$$\rightarrow \frac{1}{r-a} \rightarrow a \in (-r, r)$$

$$r+a < 0$$

$$a < -r$$

$$r-a < 0$$

$$r < a$$

$$\rightarrow \frac{1}{r-a} = \emptyset$$

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