

تابع همنی $f(x+1) + f(x+2) = 3x + f(3) \xrightarrow{x=1} f(3) + f(3) = 3 + f(3) \Rightarrow f(3) = 3$

$f(x) = x$

$y = \sqrt{f(x) - x^2} = \sqrt{-x^2 + x} \Rightarrow R = \sqrt{(-\infty, \frac{1}{4}]} \Rightarrow R = [0, \frac{1}{4}]$

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$f(x) = \frac{x^3 - 4x}{x+2} = \frac{x(x-2)(x+2)}{(x+2)} = x^2 - 2x \quad x \neq -2 \quad D_f = \mathbb{R} - \{-2\} \Rightarrow a = b = -2$

$R_f = [y_{\min}, +\infty) - \{1\} \quad \text{ext} \left| \frac{-b}{2a} = \frac{2}{-2} = -1 \Rightarrow R_f = [-1, +\infty) - \{1\} \Rightarrow C = -1$

$f\left(\frac{a+b}{2c}\right) = f(1) = 0$

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$y = \sqrt[3]{x^3 + ax^2 + bx - 1} \xrightarrow{\text{حق}} y = \sqrt[3]{(x-2)^3} \rightarrow y = \sqrt[3]{x^3 - 4x^2 + 12x - 1} \quad a = -4 \quad b = 12$

$D = [a, b] \rightarrow D = [-4, 12] \quad R = [a-2, b-2] \Rightarrow R = [-6, 10]$

$y = x - 2 \leftarrow \text{تابع}$

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$y_1 = \left(\frac{x+1}{1-x}\right)^{\frac{1}{2}} \rightarrow D_{y_1} = \mathbb{R} - \{1\} \quad y_2 = \frac{ax^2 - 1}{x^2 + b} \quad R_{y_2} = (-\infty, 1) \cup (1, +\infty)$

$x^2 \text{ متناهی} \rightarrow x^2 = \infty \rightarrow y_2 = a = 1$

$x^2 \text{ صفر} \rightarrow x^2 = 0 \rightarrow y_2 = \frac{-1}{b} = 1 \rightarrow b = -1$

$a + b = 1 - 1 = 0$

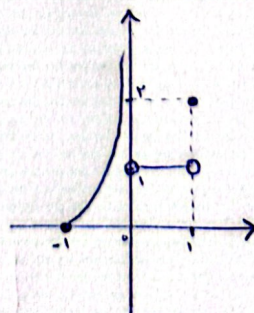
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$y = \frac{[x] + |x|}{x}$

$-1 < x < 0 \quad [x] = -1 \rightarrow y = \frac{-x-1}{x}$

$0 < x < 1 \quad [x] = 0 \rightarrow y = \frac{x}{x} = 1$

$x = 1 \rightarrow y = 2$



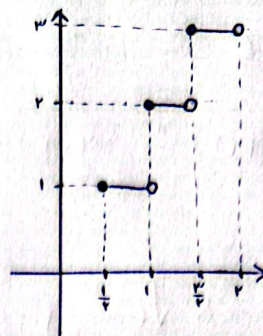
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$$y = [x] + \left[x + \frac{1}{4}\right]$$

$$\frac{1}{4} < x < 1 \quad \begin{matrix} [x] = 0 \\ [x + \frac{1}{4}] = 1 \end{matrix} \rightarrow y = 1$$

$$1 \leq x < \frac{5}{4} \quad \begin{matrix} [x] = 1 \\ [x + \frac{1}{4}] = 1 \end{matrix} \rightarrow y = 2$$

$$\frac{5}{4} \leq x < 2 \quad \begin{matrix} [x] = 1 \\ [x + \frac{1}{4}] = 2 \end{matrix} \rightarrow y = 3$$



نمودار پله خفا

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$$\left| \frac{x+1}{ax+3} \right| < 1 \quad |x+1| < |ax+3|$$

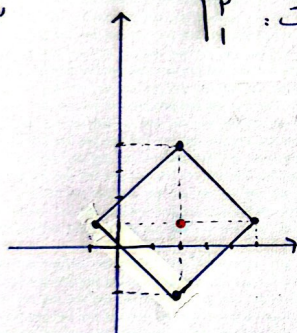
چون $\left| \frac{x+1}{ax+3} \right| < 1$ همیشه معتدلی است پس هیچ همسینه باید در این بازه از صورت بسط نماند $|a| \geq 1 \leftarrow a = 1$ یا $a = -1$ $\checkmark$ $a = -1$

$$f(x) = \left| \frac{x+1}{-x+3} \right| \quad D_f = (3, +\infty) \quad b = 3$$

$$a - b = -1 - 3 = -4$$

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$$|x-2| + |y-1| = 3$$



محور این تابع مربع شکل است که مختصات مرکز آن به این صورت است: $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ و قطر مربع برابر ۴ است.

$$x \geq 2, y \geq 1 \rightarrow x - 2 + y - 1 = 3 \quad (2, 4)$$

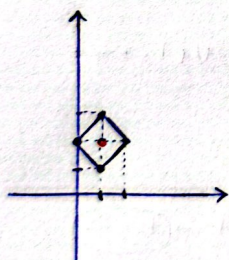
$$x \geq 2, y < 1 \rightarrow -y + 1 + x - 2 = 3 \quad (5, 1)$$

$$x < 2, y \geq 1 \rightarrow y - 1 - x + 2 = 3 \quad (2, -2)$$

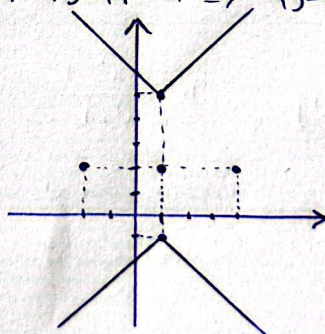
$$x < 2, y < 1 \rightarrow -y + 1 - x + 2 = 3 \quad (-1, 1)$$

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$$(الف) |x-1| + |y-2| = 1$$



$$(ب) |x-1| - |y-2| = -3 \Rightarrow |y-2| - |x-1| = 3$$



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$$y = \begin{cases} 3x + a \leq b = (a+3)x + b \\ 3x - (a+b) = (3-a)x - b \end{cases}$$

$$ax + b \geq 0$$

$$ax + b < 0$$

بسته به مثبت یا منفی

$$(a+3)(3-a) < 0 \rightarrow a+3 > 0 \quad 3-a < 0$$

$$a > -3, a > 3 \Rightarrow a > 3$$

$$a+3 < 0$$

$$a < -3$$

$$3-a > 0$$

$$a < 3$$

$$a < 3 \Rightarrow a < -3$$

$$a > 3 \leq a < -3$$

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