

$$f(rx+1) + f(x+r) = r^2x + f(r) \rightarrow a(rx+1) + b + a(x+r) + b = r^2x + r^2a + b$$

$$rax + a + b + ax + ra = r^2x + r^2a \rightarrow r^2ax + b = r^2x \rightarrow b = 0, \boxed{a=1}$$

$$\rightarrow f(x) = r^2x \rightarrow y = \sqrt{r^2x - x^2} \rightarrow \frac{-b}{ra} = \frac{-r}{-r} = \frac{r}{r} \rightarrow \sqrt{1Rf} = \sqrt{(-\infty, \frac{r}{r}]}$$

$$\rightarrow Rf: [0, \frac{r}{r}]$$

$$f(x) = \frac{x^r - rx}{x+r} \rightarrow Df: \mathbb{R} - \{a, b\} \quad Rf: [c, +\infty) - \{1\} \quad f\left(\frac{a+b}{rc}\right) = ?$$

$$\hookrightarrow \frac{x(x^r - r)}{x+r} = \frac{x(x-r)(x+r)}{(x+r)} = x^r - rx \rightarrow \frac{-b}{ra} = \frac{-r}{r} = -1 \rightarrow Rf: [1, +\infty) \rightarrow \boxed{c=1}$$

$$x^r - rx = \lambda \rightarrow x^r - rx - \lambda = 0 \quad (x-r)(x+r) = 0 \rightarrow x = -r, x = r, Df: \mathbb{R} - \{a, b\}$$

$$\Rightarrow a = -r, b = r \rightarrow f\left(\frac{a+b}{rc}\right) = \frac{-r+r}{r} = 0 \quad f(1) = \frac{1-r}{1+r} = \frac{-r}{r} = -1$$

$$y = \sqrt[3]{x^3 + ax^2 + bx - 1} \rightarrow \sqrt[3]{(x-r)^3} = x-r \quad \text{مضرب}$$

$$\rightarrow (x-r)^3 = x^3 - 9x^2 + 12x - 1 \Rightarrow \boxed{a=-9}, \boxed{b=12} \rightarrow Df: [-9, 12]$$

$$\rightarrow \max = 12 \rightarrow x-r, x=12 \rightarrow \frac{12}{12}$$

$$\rightarrow \min = -9 \rightarrow x-r, x=-9 \rightarrow \frac{-9}{-9} \rightarrow Rf: [-1, 1]$$

$$y_1 = \left(\frac{x+1}{1-x}\right)^{\frac{1}{a}} \quad y_r = \frac{ax^r - 1}{x^r + b} \quad a+b=?$$

$$D_{y_1} = \mathbb{R} - \{1\} = R_{y_r} \rightarrow R_{y_r} = \mathbb{R} - \{1\} \rightarrow ax^r - 1 \neq x^r + b \rightarrow a \neq 1$$

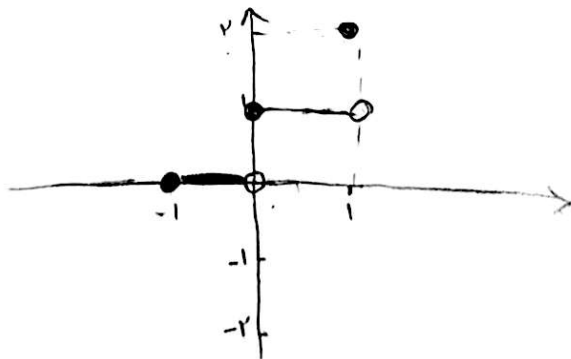
$$b \neq -1$$

$$y = \frac{[x] + |x|}{x} \rightarrow \frac{[x] + |x|}{x}$$

$$x=1 \rightarrow \frac{1}{1} = 1$$

$$-1 < x < 1 \rightarrow \frac{0+x}{x} = 1$$

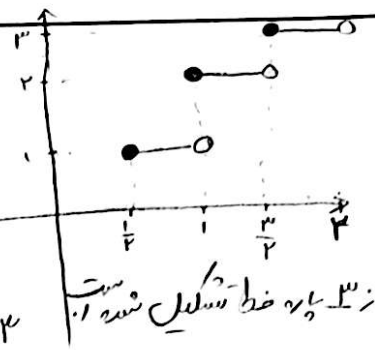
$$-1 \leq x < 0 \rightarrow \frac{-1-x}{x}$$



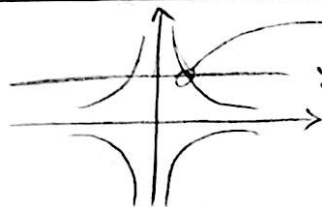
$$y = [x] + \left[x + \frac{1}{r}\right] \quad D_y: \left[\frac{1}{r}, r\right)$$

$$\frac{1}{r} \leq x < 1 \rightarrow 0 + \left[x + \frac{1}{r}\right] \rightarrow 0 + 1 = 1$$

$$1 \leq x < r \rightarrow 1 + \left[x + \frac{1}{r}\right]$$



$$y = \left| \frac{x+1}{ax+r} \right| < 1$$

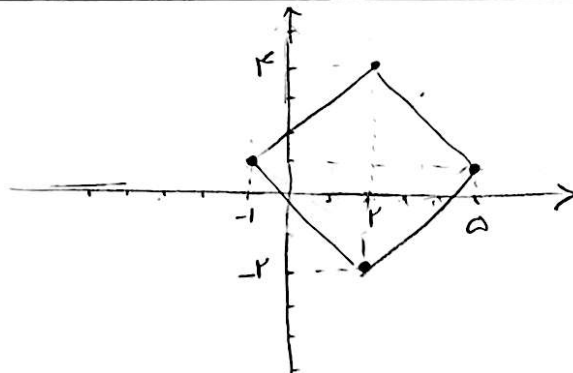


$$x+1 = ax+r \rightarrow (1-a)x = r$$

$$\frac{x+1}{ax+r} < 1 \rightarrow \frac{-\frac{r}{a}-1}{+1-1} < \frac{-\frac{r}{a}}{1}$$

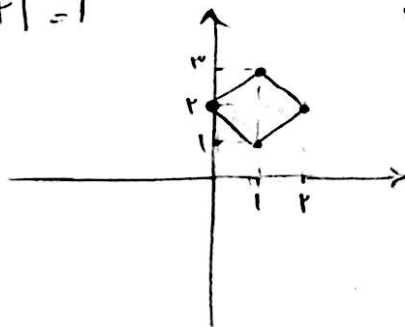
$$|x-r| + |y-1| = r$$

$$\begin{aligned} |y-1| = x+1 & \rightarrow y = x+1 \\ |y-1| = a-x & \rightarrow y = a-x \end{aligned}$$



$$|x-1| + |y-2| = 1$$

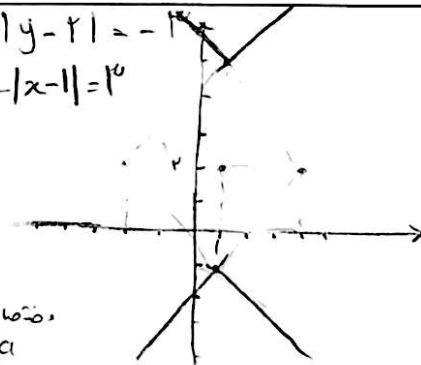
$$\begin{aligned} |x-1| &= 1-y \\ |x-1| &= y-1 \end{aligned}$$



$$|x-1| - |y-2| = -1$$

$$|x-1| - |y-2| = 1$$

$$\frac{a}{b}$$



$$y = rx + |ax+b| \quad \mathbb{R} \rightarrow \mathbb{R} \quad \textcircled{1} \quad (r-a)x+b < (r+a)x+b \rightarrow x(r-a-r-a) < 0$$

$$y = \begin{cases} (r+a)x+b & x > -\frac{b}{a} \\ rx & x = -\frac{b}{a} \\ (r-a)x+b & x < -\frac{b}{a} \end{cases}$$

$$-rax < 0 \rightarrow rax > 0 \rightarrow ax > 0$$