

$$f(x+1) + f(x+1) = 2x + f(x)$$

$$f(x) = ax + b$$

$$a(x+1) + b + a(x+1) + b = 2x + a + b$$

$$2ax + 2a + b = 2x + a + b$$

$$2a = 2 \Rightarrow a = 1 \quad b = 0$$

$$R = \sqrt{(-\infty, \frac{1}{2}]} = [-\infty, \frac{1}{2}]$$

$$y = \sqrt{-x^2 + x} \xrightarrow{\text{max } x = \frac{-b}{2a}}$$

$$\frac{-b}{2a} = \frac{-1}{-2} = \frac{1}{2} \rightarrow y = \sqrt{-\frac{1}{4} + \frac{1}{4}}$$



$$f(x) = \frac{x(x^2 - 1)}{x + 1} = \frac{x(x-1)(x+1)}{x+1} \xrightarrow{x \neq -1} x^2 - x$$

$$\xrightarrow{x = -1} x^2 - x = 1 \quad (1)^2 - (-1) = 1$$

$$R = [-1, +\infty) - \{1\}$$

$$f\left(\frac{x-1}{-1}\right) = f(-1) = (-1)^2 - (-1) = 1$$

$$x^2 - x - 1 \neq 0$$

$$(x-1)(x+1) \neq 0$$

$$\begin{matrix} x-1 \neq 0 & x+1 \neq 0 \\ x \neq 1 & x \neq -1 \end{matrix}$$

$$a = 1 \quad b = -1 \quad a = -1 \quad b = 1$$

$$y = \sqrt{x^2 + ax + b} - 1 = \sqrt{(x-1)^2} = x-1$$

$$\sqrt{x^2 - 4x + 12} - 1$$

$$a = -4 \quad b = 12$$

$$D = [-4, 12]$$

$$\Rightarrow -4 \leq x \leq 12 \quad -1 \leq x-1 \leq 11 \rightarrow R = [-1, 11]$$

$$y_1 = \left(\frac{x+1}{1-x}\right)^{\frac{1}{2}} \rightarrow y_1 = \sqrt{\frac{x+1}{-x+1}} \xrightarrow{\text{مخرج ثابت}} D = R - \{1\}$$

$$y_1 = \frac{ax^2 - 1}{x^2 + b} \rightarrow y_1 x^2 + b y_1 = ax^2 - 1$$

$$(y_1 - a)x^2 = -1 - b y_1$$

$$x^2 = \frac{-1 - b y_1}{y_1 - a} \Rightarrow -b = \frac{1}{a} \Rightarrow ab = -1$$

$$\begin{matrix} a = 1 & b = -1 \\ a = -1 & b = 1 \end{matrix}$$

$$-1 \leq x < 0 \rightarrow [x] = -1 \quad |x| = -x$$

$$y = [x] + \frac{|x|}{x} \rightarrow x \neq 0$$

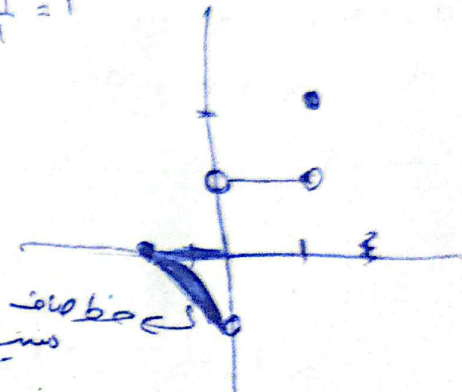
$$\frac{[x]}{x} + \frac{|x|}{x} = \frac{-1}{x} = -1$$

$$0 \leq x < 1 \rightarrow [x] = 0 \rightarrow |x| = x$$

$$x = 1 \rightarrow \frac{1}{1} + \frac{1}{1} = 2$$

$$\frac{[x]}{x} + \frac{|x|}{x} = 1$$

مسبب جیوں $-\frac{1}{x}$ کا



$$y = [x] + [x + \frac{1}{x}]$$

$$\frac{1}{x} \leq x < 1 \rightarrow [x] = 0$$

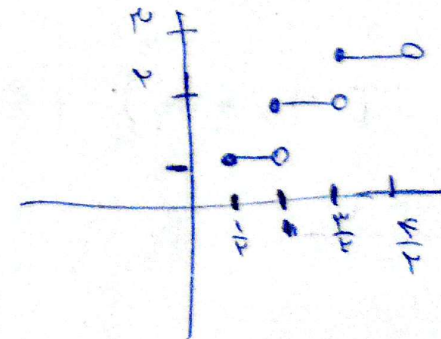
$$\frac{1}{x} \leq x + \frac{1}{x} < \frac{2}{x} \rightarrow [x + \frac{1}{x}] = 1 \rightarrow y = 0 + 1 = 1$$

$$1 \leq x < \frac{2}{x} \rightarrow [x] = 1$$

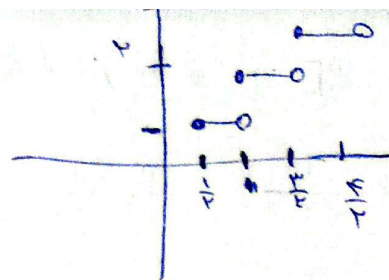
$$\frac{1}{x} \leq x + \frac{1}{x} < \frac{2}{x} \rightarrow [x + \frac{1}{x}] = 1 \rightarrow y = 1 + 1 = 2$$

$$\frac{2}{x} \leq x < 2 \rightarrow [x] = 1$$

$$2 \leq x + \frac{1}{x} < \frac{3}{x} \rightarrow [x + \frac{1}{x}] = 2$$



$$\begin{aligned}
 & \frac{1}{2} \leq x + \frac{1}{4} < \frac{3}{4} \rightarrow [x+1] = 1 \rightarrow y = 0+1=1 \\
 & 1 \leq x < \frac{5}{4} \rightarrow [x] = 1 \rightarrow y = 1+1=2 \\
 & \frac{3}{4} \leq x + \frac{1}{4} < \frac{5}{4} \rightarrow [x+1] = 1 \rightarrow y = 1+1=2 \\
 & \frac{5}{4} \leq x < 2 \rightarrow [x] = 1 \rightarrow y = 1+1=2 \\
 & 2 \leq x + \frac{1}{4} < \frac{9}{4} \rightarrow [x+1] = 2 \rightarrow y = 1+2=3
 \end{aligned}$$



$$y = \left| \frac{x+1}{ax+b} \right| < 1 \xrightarrow{\text{D.O.B.}} \frac{(x+1)^2}{(ax+b)^2} < 1 \Rightarrow (x+1)^2 < (ax+b)^2$$

$$a^2 = 1 \xrightarrow{\beta_1} a = 1 \rightarrow x^2 + 2x + 1 < x^2 + 4x + 4 \rightarrow -2 < 2x \rightarrow x > -1 \rightarrow (-1, +\infty)$$

$$\begin{aligned}
 & \xrightarrow{\beta_1} a = -1 \rightarrow x^2 + 2x + 1 < x^2 - 4x + 4 \rightarrow 1x < 1 \rightarrow x < 1 \\
 & D = (-\infty, 1) \text{ دایره }
 \end{aligned}$$

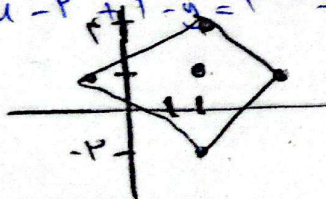
$$\rightarrow a=1, b=-2 \rightarrow a-b=1-(-2)=3$$

$$\begin{aligned}
 & x-2=y-1 \rightarrow y=x-1 \\
 & |x-2|+|y-1|=3
 \end{aligned}$$

در این مسئله، دو خط موازی داریم که به هم می‌رسند.

$$\begin{aligned}
 x > 2, y < 1 & \rightarrow x-2+y-1=3 \rightarrow y = -x+5 \\
 x < 2, y > 1 & \rightarrow 2-x+y-1=3 \rightarrow y = x+2 \\
 x < 2, y < 1 & \rightarrow 2-x-y+1=3 \rightarrow y = -x \\
 x > 2, y > 1 & \rightarrow x-2+y-1=3 \rightarrow y = -x+5
 \end{aligned}$$

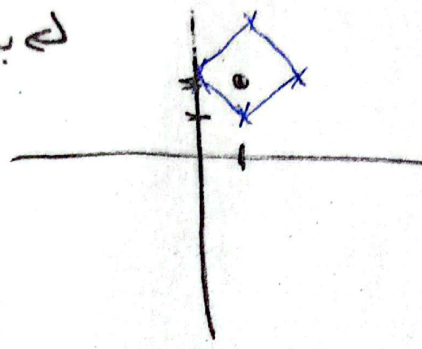
$x=5 \rightarrow y=0$
 $x=2 \rightarrow y=2$
 $x=-2 \rightarrow y=2$
 $x=-1 \rightarrow y=1$
 $x=2 \rightarrow y=-2$



الف) $|x-1| + |y-2| = 1$

به طرف

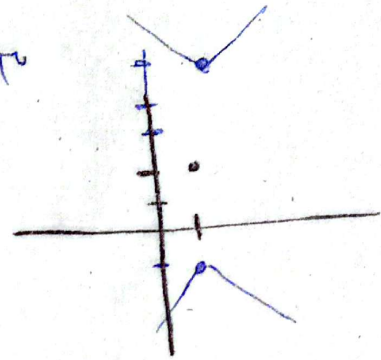
$x=1$
 $y=2$



نمودار مربع

ب) $|x-1| + |y-2| = -1$

$x=1$
 $y=2$

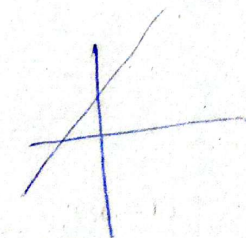


نمودار از این سمت کو مختار

$y = kx + |ax + b|$

$ax + b = 0$
 $ax = -b \rightarrow x = -\frac{b}{a}$

$kx - ax - b$	$-\frac{b}{a}$	$kx + ax + b$
$(k-a)x - b$	$-\frac{b}{a}$	$(k+a)x + b$



$ka > 0$
 $a > -k$
 $k - a > 0$
 $k > a$

$ka < 0$
 $a < -k$
 $k - a < 0$
 $k < a$

استوak ناله سوس $\emptyset$