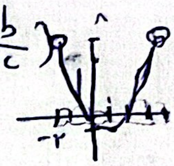
زرین [illegible] دوازدهم A

① $f(2x+1)+f(x+2)=3x+f(3) \xrightarrow{x=1} f(3)+\cancel{f(3)}=3x+f(3)$

$f(3)=3x \longrightarrow y=\sqrt{f(x)-x^2} \xrightarrow{x=3} y=\sqrt{3x-9} \longrightarrow R(f)=[0,+\infty)$

② $f(x)=\dfrac{x^3-4x}{x+2}$ $\quad D_f=\mathbb{R}-\{a,b\}$ $\quad R_f=[c,+\infty)-\{8\}$ $\quad f\left(\dfrac{a+b}{2c}\right)$



$f(x)=\dfrac{x(x-2)(x+2)}{\cancel{x+2}} \xrightarrow{x\neq -2} f(x)=x(x-2) \longrightarrow f(x)=x^2-2x$

$\left| -\dfrac{b}{2a}=\dfrac{2}{2}=1\right.$

$R_f=[-1,+\infty)-\{8\} \xrightarrow[\text{نباید جزء دامنه باشه}]{\text{در ۴ هم}} \longrightarrow D_f=\mathbb{R}-\{-2,4\} \longrightarrow \boxed{a,b=-2,4}$

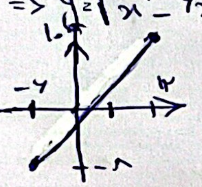
$\boxed{C=-1}$

$f\left(\dfrac{2}{-2}\right)=f(-1)=\dfrac{-1+4}{1}=\boxed{3}$

③ $y=\sqrt[3]{x^3+ax^2+bx-8}$ $\quad D_f=[a,b]$ $\quad R_f=?$

$y=\sqrt[3]{(x-2)^3} \Rightarrow y=\sqrt[3]{x^3-6x^2+12x-8} \longrightarrow a=-6 \quad b=12 \longrightarrow D_f=[-6\ \ 12]$

$y=x-2$



$R_f=[-8\ \ 10]$

④ $y_1=\left(\dfrac{x+1}{1-x}\right)^{\frac{1}{5}} \longrightarrow x\neq +1$ $\qquad y_2=\dfrac{ax^2-1}{x^2+b}\neq 1 \longrightarrow \dfrac{a}{1}=1 \longrightarrow a=1$

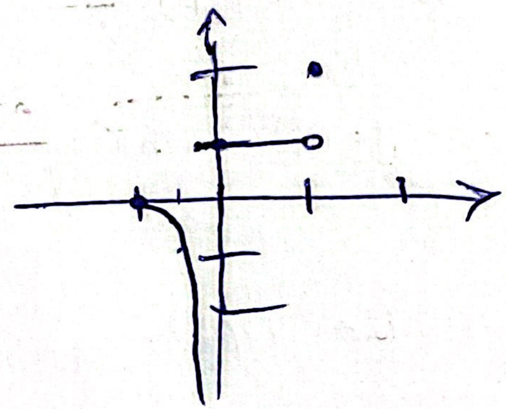
$\dfrac{x^2-1}{x^2-1}$

a) $y = \dfrac{[x]}{x} + \dfrac{|x|}{x}$

$-1 \le x < 0 \longrightarrow y = \dfrac{-1}{x} + (-1) = \dfrac{-1-x}{x}$

$0 \le x < 1 \longrightarrow y = 0 + 1$

$x = 1 \qquad y = 2$

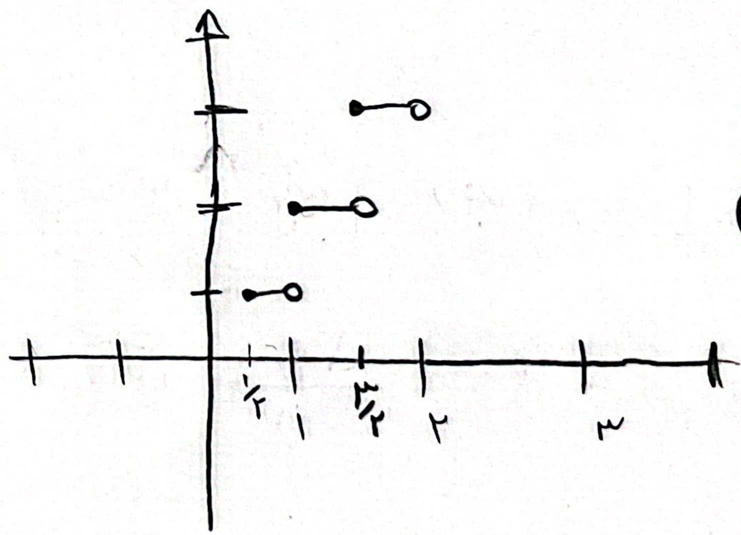


b) $y = [x] + [x + \frac{1}{2}] \qquad [\frac{1}{2}, 2)$

$\frac{1}{2} \le x < 1 \longrightarrow y = 0 + 1 = 1$

$1 \le x < \frac{3}{2} \longrightarrow y = 1 + 1 = 2$

$\frac{3}{2} \le x < 2 \longrightarrow y = 1 + 2 = 3$



(۱ و ۲ و ۳)

(1) $\left|\frac{x+1}{ax+3}\right| < 1$

$(x+1)^2 < (ax+3)^2$

$x^2+2x+1 < a^2x^2+6ax+9$

$(1-a^2)x^2+(2-6a)x-8<0$

$a=1 \rightarrow -4x-8<0 \qquad x+2>0$

طول $\boxed{(-2,+\infty)}$

$a=-1 \rightarrow 8x-8<0 \qquad x-1<0$

$x<1 \quad (-\infty, 1) \quad \alpha$

طول $\boxed{=1-(-2)=3}$

$\Delta>0 \rightarrow (2-6a)^2+32(1-a^2)>0$

$36a^2-24a+4+32-32a^2>0$

$4a^2-24a+36>0$

$a^2-6a+9>0$

$(a-3)^2>0 \rightarrow \boxed{a\neq 3}$

$x-1$

⑧ $|x-۲|+|y-۱|=۳$

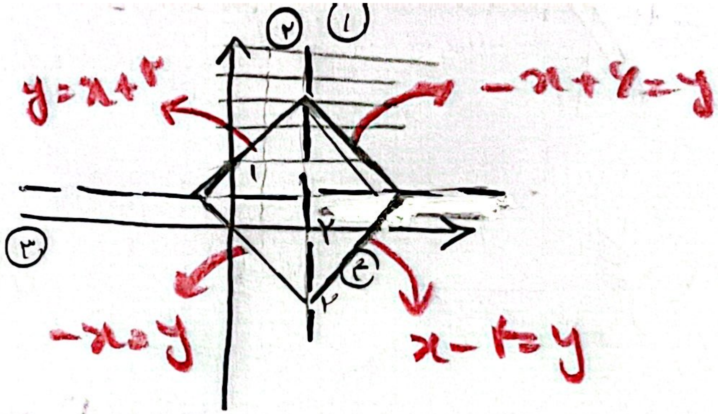
① $\rightarrow x-۲+y-۱=۳$

$y=-x+۶$

② $-(x-۲)+(y-۱)=۳$

$-x+۲+y-۱=۳$

$y=x+۲$



③ $-(x-۲)-(y-۱)=۳$

$-x+۲-y+۱=۳$

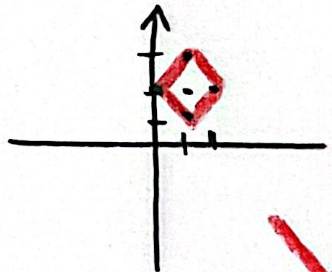
$-x=y$

④ $(x-۲)-(y-۱)=۳$

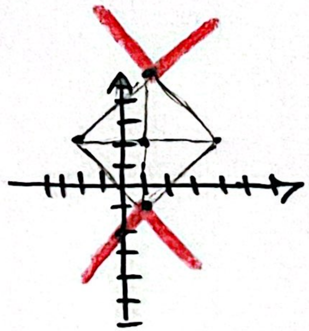
$x-۲-y+۱=۳$

$x-۴=y$

⑨ الف) $|x-۱|+|y-۲|=۱$



ب) $|x-۱|-|y-۲|=-۳$

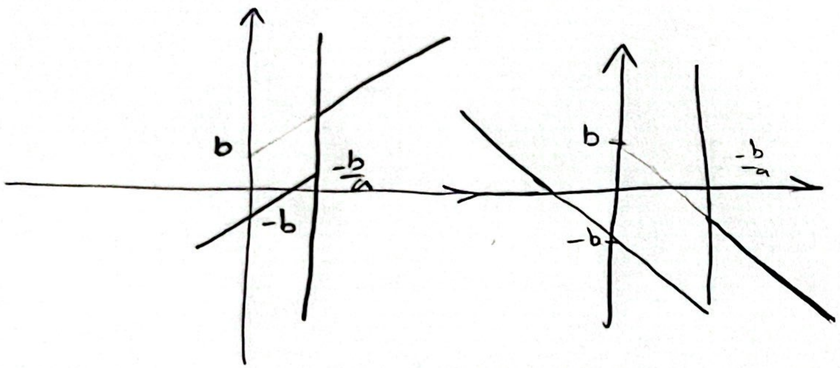


(10) $y = 3x + |ax + b|$

$$-\frac{b}{a}$$

$y = 3x - ax - b$ | $y = 3x + ax + b$

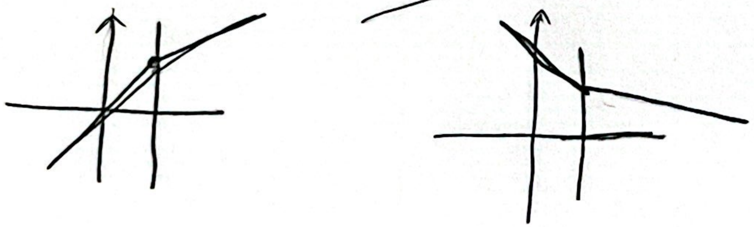
$y = x(3-a) - b$ $\qquad$ $y = x(3+a) + b$



α

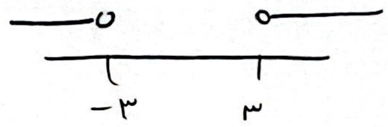
$$-\frac{b}{a}(3-a) - b = -\frac{b}{a}(3+a) + b$$

$$-\frac{3b}{a} + \cancel{b} - \cancel{b} = -\frac{3b}{a} \cancel{-b} + \cancel{b}$$ $\longrightarrow$ در محل ریشه قدر مطلق ۲ تابع برابر می‌شوند



$\longrightarrow$ $\left.\begin{array}{ll} 3+a>0 & -3<a \\ 3-a>0 & 3>a \end{array}\right\} \longrightarrow -3<a<3$

$\longrightarrow$ $\left.\begin{array}{ll} 3+a<0 & a<-3 \\ 3-a<0 & 3<a \end{array}\right\} \longrightarrow \emptyset$



$\longrightarrow$ $a = (-3 \quad 3)$