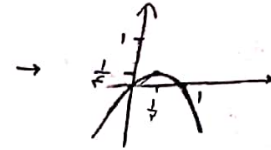


$$f(x(x+1)) + f(x(x+2)) = 3x + f(3) \rightarrow x=1 \Rightarrow f(2) + f(3) = 3 + f(3) \Rightarrow f(2) = 3$$

از آنجایی که تابع خطی است و همواره برقرار است پس $f(x) = x$ است

$$y = \sqrt{f(x) - x^2} \xrightarrow{f(x)=x} y = \sqrt{x - x^2} \rightarrow x - x^2 = x(1-x), \text{ ext } \left| \frac{-b}{2a} \right| = \frac{1}{2}$$



$$\rightarrow R = (-\infty, \frac{1}{2}] \quad \text{چون برای اعداد مثبت و اعداد منفی داریم} \quad R = [0, \frac{1}{2}] \quad \checkmark$$

(۲)

$$f(x) = \frac{x^3 - 4x}{x+2} \quad \left\{ \begin{array}{l} x+2 \neq 0 \rightarrow x \neq -2 \rightarrow \boxed{\alpha \neq -2} \end{array} \right.$$

$$D_f = \mathbb{R} - \{\alpha, b\}$$

$$\boxed{b = 4} \leftarrow (x+4), (x \neq -2)$$

$$R_f = [6, +\infty) - 1 \rightarrow \frac{x^3 - 4x}{x+2} \neq 1 \rightarrow \frac{x(x-2)(x+2)}{x+2} \Rightarrow x(x-2) \neq 1 \rightarrow x^2 - 2x - 1 \neq 0 \rightarrow (x-1)(x+3) \neq 0$$

$$y = \frac{x^3 - 4x}{x+2} \rightarrow \frac{x(x-2)(x+2)}{x+2} = x^2 - 2x \rightarrow \text{ext } \left| \frac{-b}{2a} \right| = \frac{-2}{2} = -1 \quad \left\{ \begin{array}{l} R_f = [-1, +\infty) - \{1\} \Rightarrow \boxed{C = -1} \end{array} \right.$$

(۲)

$$y = \sqrt[3]{x^3 + ax^2 + bx - 1} \xrightarrow{\text{فرض کنیم برای هر } x \text{ برقرار است}} y = \sqrt[3]{(x-2)^3} \Rightarrow (x-2)^3 = x^3 - 6x^2 + 12x - 8 \rightarrow \boxed{\alpha = -6}, \boxed{b = 12}$$

$$\Rightarrow D_f = [-6, 12]$$

$$\text{if } x = -6 \rightarrow y = \sqrt[3]{(-6-2)^3} = -8 - 2 = -10 \quad \text{if } x = 12 \rightarrow y = \sqrt[3]{(12-2)^3} = 10 - 2 = 8 \quad \left\{ \begin{array}{l} R_f = [-10, 8] \end{array} \right. \quad \checkmark$$

(۲)

$$y_1 = \left(\frac{x+1}{1-x} \right)^{\frac{1}{2}} = \sqrt{\frac{x+1}{1-x}} \quad \text{فرض کنیم برای هر } x \text{ برقرار است} \rightarrow 1-x \neq 0 \rightarrow x \neq 1 \rightarrow D_f = \mathbb{R} - \{1\} \rightarrow R_f = \mathbb{R} - \{1\}$$

$$y_2 = \frac{ax^2 - 1}{x^2 - b} \rightarrow x^2 = \frac{-by_2 + 1}{y_2 - a} = \frac{by_2 - 1}{y_2 - a} \rightarrow y_2 - a \neq 0, y_2 \neq 1 \rightarrow \boxed{\alpha = 1} \rightarrow y_2 = \frac{x^2 - 1}{x^2 - b}$$

$$\rightarrow y_2 \neq 1 \rightarrow \frac{x^2 - 1}{x^2 - b} \neq 1 \rightarrow x^2 - b \neq x^2 - 1 \rightarrow \boxed{b = 1} \rightarrow \alpha + b = 1 + 1 = 2 \quad \checkmark$$

(۲)

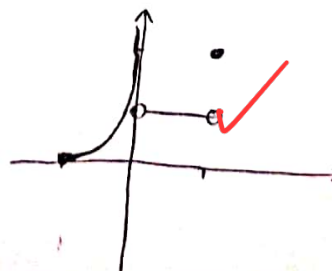
$$y = \frac{[x]}{x} + \frac{|x|}{x}$$

$$-1 < x < 0 \rightarrow [x] = -1, \frac{|x|}{x} = -1 \rightarrow y = \frac{-1}{x} - 1 \xrightarrow{x=-1} 1 - 1 = 0, x=0^- \rightarrow y = +\infty$$

$$x = 0 \rightarrow \text{تعریف نشده}$$

$$0 < x < 1 \rightarrow [x] = 0, \frac{|x|}{x} = 1 \rightarrow \boxed{y = 1}$$

$$x = 1 \rightarrow y = 1 + 1 = 2$$



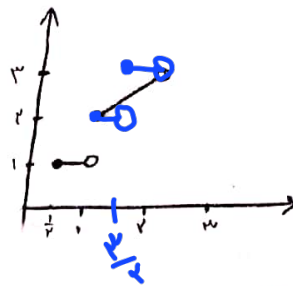
(۲)

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$$y = [x] + [x + \frac{1}{2}]$$

$$\frac{1}{2} < x < 1 \rightarrow [x] = 0 \begin{cases} x = \frac{1}{2} \rightarrow 0 + [1] = 1 \\ x = 1 \rightarrow 0 + [\frac{3}{2}] = 1 \end{cases}$$

$$1 \leq x < 2 \rightarrow [x] = 1 \begin{cases} x = 1 \rightarrow 1 + [\frac{3}{2}] = 2 \\ x = 2 \rightarrow 1 + [\frac{5}{2}] = 3 \end{cases}$$



از باره خط تشکیل شده است.

⑨

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$$|x-2| + |y-1| = 3$$

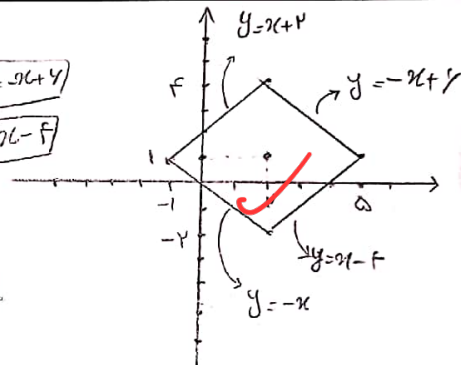
⑤ ① $\rightarrow x-2 + |y-1| = 3 \rightarrow |y-1| = -x+5$

$$\text{if } x > 2 \rightarrow x-2 + |y-1| = 3 \rightarrow |y-1| = -x+5$$

$$\begin{cases} y-1 = -x+5 \Rightarrow y = -x+6 \\ y-1 = -(-x+5) \Rightarrow y = x-4 \end{cases}$$

if $x < 2 \rightarrow 2-x + |y-1| = 3 \rightarrow |y-1| = x+1$

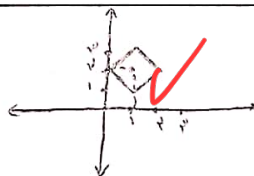
$$\begin{cases} y-1 = -(x+1) \Rightarrow y = -x \\ y-1 = +(x+1) \Rightarrow y = x+2 \end{cases}$$



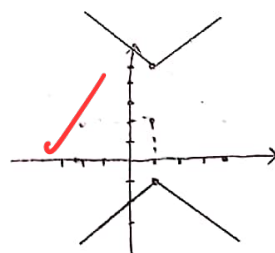
②

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الف) $|x-1| + |y-2| = 1$



ب) $|x-1| - |y-2| = -3 \rightarrow |y-2| - |x-1| = 3$



④

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$$y = 3x + |ax+b| \xrightarrow{\text{با } x \text{ ضرب و جمع}} y = 3x + ax + b = y = (3+a)x + b \xrightarrow{\text{با } x \text{ ضرب و جمع}} 3+a \neq 0 \rightarrow a \neq -3$$

$$y = 3x + |ax+b| \xrightarrow{\text{با } x \text{ ضرب و جمع}} y = 3x - ax - b = y = (3-a)x - b \xrightarrow{\text{با } x \text{ ضرب و جمع}} 3-a \neq 0 \rightarrow a \neq 3$$

$$\alpha = \mathbb{R} - \{-3, 3\}$$

ممنوع است که $\alpha = -3$ یا $\alpha = 3$ باشد. $\alpha \in \mathbb{R}$ باشد. $-3 < \alpha < 3$

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