

$$f(x+1) + f(x-1) = 2x + f(x) \quad \text{خط لایه } \rightarrow y = ax + b$$

$$\left. \begin{aligned} f(x+1) &= a(x+1) + b = ax + a + b \\ f(x-1) &= a(x-1) + b = ax - a + b \end{aligned} \right\} \rightarrow 2ax + 2b$$

$$2ax + 2b = 2x + a + b \rightarrow 2a = 2 \rightarrow a = 1 \quad \rightarrow y = ax + b \rightarrow y = x$$

$$2b = a + b \rightarrow b = 1$$

$$y = \sqrt{f(x) - x^2} = \sqrt{x - x^2}$$

$$\left| \begin{aligned} \frac{dy}{dx} &= \frac{1}{2\sqrt{x-x^2}} \\ \frac{1}{2\sqrt{x-x^2}} &= \frac{1}{2} \end{aligned} \right.$$

$$R_f = \left[-\frac{1}{2}, \frac{1}{2} \right] = \left[0, \frac{1}{2} \right]$$

$$f(x) = \frac{x^3 - f(x)}{x+1} = \frac{x(x-1)(x+1)}{x+1} = x(x-1) = x^2 - x$$

$$\left| \begin{aligned} \frac{dy}{dx} &= \frac{2x-1}{2\sqrt{x-x^2}} = 1 \\ 2x-1 &= 2\sqrt{x-x^2} \end{aligned} \right.$$

$$\rightarrow R_f = [-1, 1] - \{0\} \quad \text{خط لایه } \rightarrow x(x-1) = 1 \rightarrow x = \frac{1}{2} \rightarrow D_f = \mathbb{R} - \left\{ \frac{1}{2} \right\}$$

$$f\left(\frac{a+b}{2}\right) = f\left(\frac{1+(-1)}{2}\right) = f(0) = \frac{(-1)^3 - f(-1)}{-1+1} = -1+1 = 0$$

$$y = \sqrt{x^3 + ax^2 + bx - 1} = \sqrt{(x-1)^3} = \sqrt{x^3 - 3x^2 + 3x - 1}$$

$$D_f = [a, b] = [-4, 1] \quad y = \sqrt{(x-1)^3} = x-1$$

$$-1 \leq y \leq 1 \quad \rightarrow R_f = [-1, 1]$$

$$y_1 = \left(\frac{x+1}{1-x}\right)^2 \quad 1-x \neq 0 \rightarrow x \neq 1$$

$$D_{y_1} = \mathbb{R} - \{1\} \rightarrow R_{y_1} = \mathbb{R} - \{1\}$$

$$\left| \begin{aligned} y_1 &= 1 \rightarrow \frac{x+1}{1-x} = 1 \rightarrow x+1 = 1-x \rightarrow x = 0 \\ y_2 &= -1 \rightarrow \frac{x+1}{1-x} = -1 \rightarrow x+1 = -1+x \rightarrow x = -2 \end{aligned} \right. \rightarrow a+b = 1+1-1 = 1$$

$$y = \frac{[x]}{x} + \frac{[x]}{x}$$

$$-1 \leq x < 0 \rightarrow \frac{[x]}{x} = -1$$

$$0 \leq x < 1 \rightarrow \frac{[x]}{x} + 1 = 0 + 1 = 1$$

$$x = 1 \rightarrow 1+1 = 2$$



مسائل

$$y = [x] + [x + \frac{1}{2}] = [2x]$$

$$\frac{1}{2} < x < 1$$

$$1 < x < \frac{3}{2}$$

$$\frac{3}{2} < x < 2$$



بداية 1/2

$$\left| \frac{x+1}{ax+b} \right| = 1 \Rightarrow x+1 = \pm(ax+b) \Rightarrow x(1-a) = \mp b \Rightarrow x = \frac{\mp b}{1-a}$$

$$\left| \frac{x+1}{ax+b} \right| = -1 \Rightarrow x+1 = \pm(ax+b) \Rightarrow x(1+a) = \mp b \Rightarrow x = \frac{\mp b}{1+a}$$

$$\text{مثال } y=1 \text{ } b=1 \text{ } a=1 \Rightarrow \left| \frac{x+1}{x+1} \right| = 1 \Rightarrow x=b \Rightarrow x=1$$

$$b=1 \Rightarrow x=-1 \Rightarrow -1 = \frac{-b}{1+a} \Rightarrow -1-a = -1 \Rightarrow a=0 \quad a-b = 0-1 = -1$$

$$|x-1| + |y-1| = 2$$

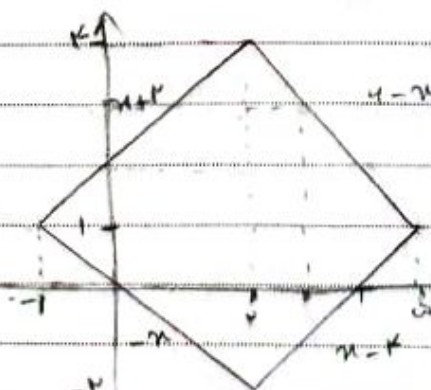
$$|y-1| = 2 - |x-1|$$

$$x > 1 \Rightarrow |y-1| = 2 - (x-1) = 3-x \Rightarrow |y-1| = 3-x \Rightarrow y-1 = 3-x \Rightarrow y = 4-x$$

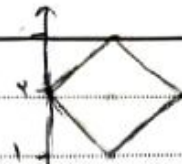
$$y-1 = x-2 \Rightarrow y = x-1$$

$$x < 1 \Rightarrow |y-1| = 2 - (1-x) = 1+x \Rightarrow |y-1| = 1+x \Rightarrow y-1 = 1+x \Rightarrow y = x+2$$

$$y-1 = -1-x \Rightarrow y = -x$$



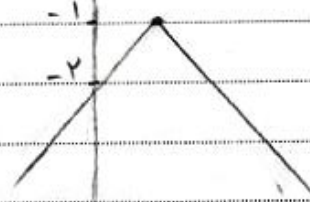
الف) $|x-1| + |y-2| = 1$



ب) $|x-1| - |y-2| = -3$



$\Rightarrow |y-2| - |x-1| = 3$



$y = mx + |cn + b|$

$mx - cn - b = (m-a)x - b$

$\frac{b}{a}$

$mx + cn + b = (m+a)x + b$

برای اینکه $R \geq 2$ باشد باید $|a| \geq 1$ و $|b| \geq 1$ باشد.