تمرين (تدريب)

$$y = -\frac{1}{2}\left(x^2 - \frac{8}{3}x - \frac{1}{3}\right)^{\frac{3}{2}} + \frac{8}{5}x + 1 = -\frac{1}{2}\left(x^2 - \frac{8}{3}x - \frac{1}{3}\right)^{\frac{3}{2}} + \frac{8}{5}x + 1$$

$$\frac{dy}{dx} = -\frac{1}{2}\left(\frac{2}{3}x - \frac{8}{9}\right)\left(x^2 - \frac{8}{3}x - \frac{1}{3}\right)^{\frac{1}{2}} = \left(\frac{4}{9} - \frac{2}{3}x\right)\left(x^2 - \frac{8}{3}x - \frac{1}{3}\right)^{\frac{1}{2}}$$

$$\Rightarrow y = -\frac{1}{2}\left(\frac{2}{3} - x\right)\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = \left(\frac{2}{3} - x\right)\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{\frac{1}{2}}\left(\frac{2}{3} - x\right)\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = \left(\frac{2}{3} - x\right)^2\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{\frac{1}{2}}\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{\frac{1}{2}}$$

$$\frac{d^3y}{dx^3} = 2\left(\frac{2}{3} - x\right)\left(-1\right)\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{\frac{1}{2}} + \left(\frac{2}{3} - x\right)^2\left(\frac{8}{9} - 2x\right)\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{-\frac{1}{2}}$$

$$\frac{d^4y}{dx^4} = -2\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{\frac{1}{2}} + 2\left(\frac{2}{3} - x\right)\left(\frac{8}{9} - 2x\right)\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{-\frac{1}{2}} + 2\left(\frac{2}{3} - x\right)\left(\frac{8}{9} - 2x\right)\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{-\frac{1}{2}} + \left(\frac{2}{3} - x\right)^2(-2)\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{-\frac{3}{2}}$$

$$\frac{d^4y}{dx^4} = -2\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{\frac{1}{2}} + 4\left(\frac{2}{3} - x\right)\left(\frac{8}{9} - 2x\right)\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{-\frac{1}{2}} - 2\left(\frac{2}{3} - x\right)^2\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{-\frac{3}{2}}$$

$$\frac{d^4y}{dx^4} + 2\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{\frac{1}{2}} = \left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{-\frac{3}{2}}\left(-4\left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^2 + 4\left(\frac{2}{3} - x\right)\left(\frac{8}{9} - 2x\right)\left(\frac{4}{9} + \frac{8}{9}x - x^2\right) - 2\left(\frac{2}{3} - x\right)^2\right)$$

$$\frac{d^4y}{dx^4} + 2y = \left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{-\frac{3}{2}}\left(-\frac{64}{81} - \frac{256}{81}x - \frac{256}{81}x^2 + \frac{64}{81}x^3 + \frac{256}{81}x^2 - \frac{64}{81}x - 2\left(\frac{4}{9} - \frac{4}{3}x + x^2\right)\right)$$

$$\frac{d^4y}{dx^4} + 2y = \left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{-\frac{3}{2}}\left(-\frac{64}{81} - \frac{192}{81}x + \frac{64}{81}x^3 - \frac{8}{9} + \frac{24}{9}x - 2x^2\right)$$

$$\frac{d^4y}{dx^4} + 2y = \left(\frac{4}{9} + \frac{8}{9}x - x^2\right)^{-\frac{3}{2}}\left(\frac{64}{81}x^3 - 2x^2 + \frac{24}{81}x - \frac{136}{81}

$D_f = [-1, 2]$

$R_f = (-\infty, 2]$

$D_f = [-2, 1]$

$R_f = (-\infty, 3]$

$(-1, 4)$

$D_f = [-1, 1/2]$

$(3, -1/2)$

$D_f = [-1, 1/2]$

$R_f = [-1, +\infty)$

$(-1/2, -1)$

$R_f [-1, +\infty)$

$(-1/2, -1)$

$\alpha\beta = -1 \quad \alpha - 1/2 = 1/2$

$$\sqrt{2x} \longrightarrow \sqrt{2x+1}$$

$$\longrightarrow -2 + \sqrt{2x+1} \longrightarrow 2 + \sqrt{2x+1}$$

$$\frac{8}{8} = \sqrt{2x+1} \qquad \frac{7}{8} = 2 + \sqrt{2x+1}$$

$$\Rightarrow \frac{4}{4} = 2x+1 \Rightarrow \frac{3}{4} = 2x$$

$$\Rightarrow x = \frac{3}{2}$$

$D_f = (a, f] \rightarrow D_{f(x-1)} = (a+1, f]$

$D_g = [-1, b) \rightarrow D_{g(x+1)} = [-2, b-1)$

$a+1 = -2 \Rightarrow a = -3$

$b-1 = 0 \Rightarrow b = 1$

$\Rightarrow -3-1 = -4 = a-b$

$D = [1/2, 3/2]$

$R = [0, 2]$

$x \to [ax]$ $x = 0$

a ← یعنی $x \Rightarrow -a$

$\frac{-a}{|a|}$

$[ax < 1 \Rightarrow a < 0$

$x = \frac{1}{|a|} \to \left[\frac{1}{|a|}\right] a^{-1} = \frac{1}{a^{-1}} = +\frac{1}{a}$

$x^{+} = \frac{1}{+|a|} \to [a \cdot |a|^{-}] a^{-1} = \frac{1}{a^{-1}} = +\frac{1}{a}$

-2

$\frac{2}{a^2} = \frac{1}{3}$ $\quad a^2 = 1 \Rightarrow a = -\sqrt{2}$

$b = \frac{2}{|a|} = \frac{1}{\sqrt{2}}$ $\quad b - a = \frac{1}{\sqrt{2}} + \sqrt{2}$

$\Rightarrow \sqrt{2}\left(a + \frac{1}{2}\right) = \frac{3\sqrt{2}}{2}$ $\quad \frac{3\sqrt{2}}{2}$

$$f(x) = \log_{\frac{1}{2}} x$$

$$\log_{\frac{1}{2}}(x+2) - \log_{\frac{1}{2}}(x-1) = \log_{\frac{1}{2}}(x^2+x-2)$$

$$\log_{\frac{1}{2}}(x+2) - \log_{\frac{1}{2}}(x-1) = \log_{\frac{1}{2}}\frac{x+2}{x-1}$$

$$\log_{\frac{1}{2}}\frac{x+2}{x-1} \geq -1$$

$$\frac{x+2}{x-1} \leq \left(\frac{1}{2}\right)^{-1}$$

$$\frac{x+2}{x-1} \leq 2$$

$$\frac{x+2}{x-1} - 2 \leq 0$$

$$\frac{x+2-2x+2}{x-1} \leq 0$$

$$\frac{-x+4}{x-1} \leq 0$$

$$\frac{x-4}{x-1} \geq 0$$

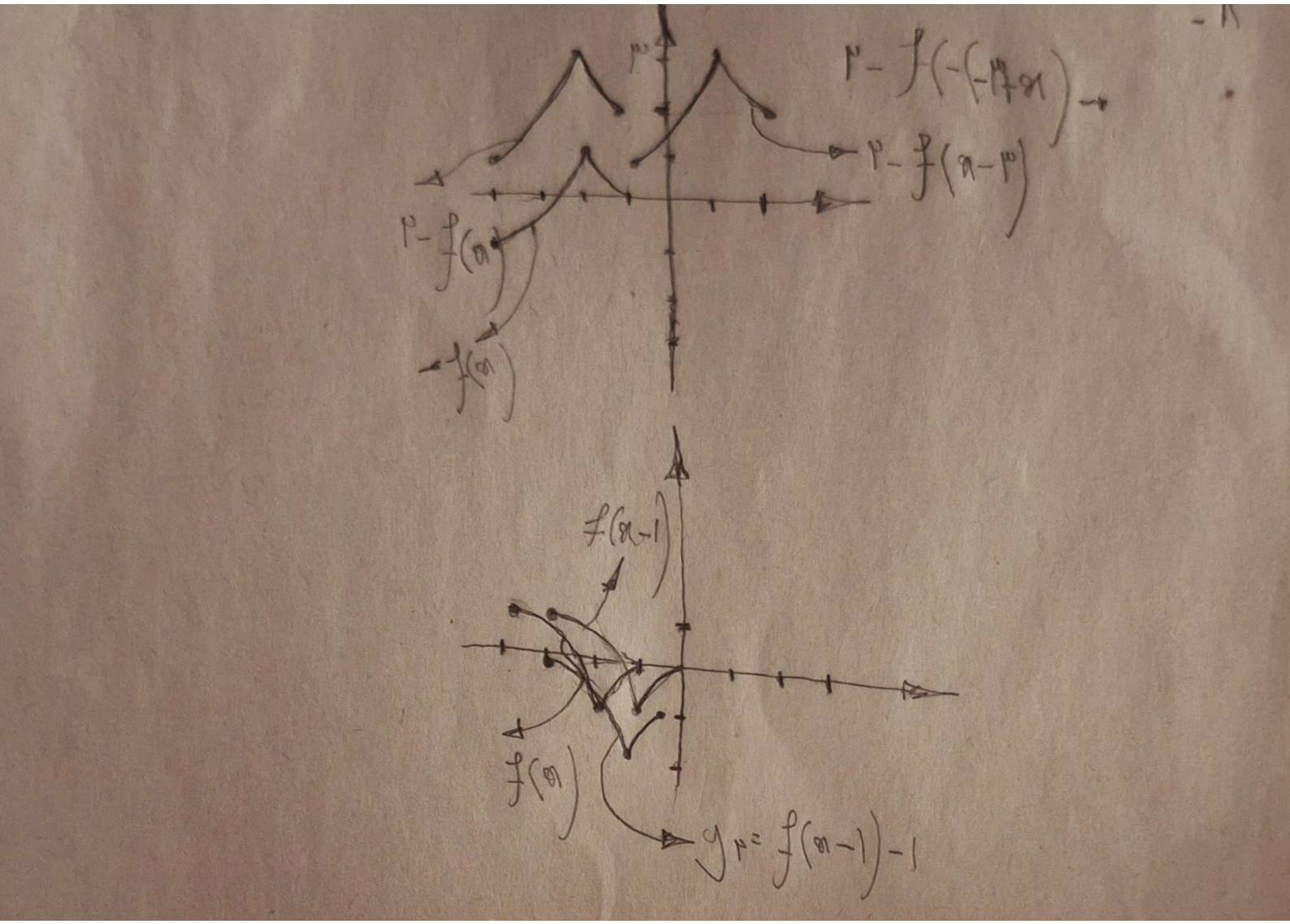
$$x-4 = 0 \quad x-1 = 0$$

$$x = 4 \quad x = 1$$

$$(x-4)(x-1) \geq 0$$

$$x^2 - 4x - x + 4 \geq 0$$

$$x^2 - 5x + 4 \geq 0$$


y = f(x-1)
y = f(x)
y = f(x-1)-1

$$f(x) = \left|\frac{x+1}{x+2}\right| \longrightarrow \left|\frac{x+1}{x+2} - 2\right| \longrightarrow$$

$$= \left|\frac{x+1}{x+2} + \frac{x+2}{x+2}\right| = \left|\frac{2x+3}{x+2}\right| \longrightarrow$$

$$g(x) = \left|\frac{2x+3}{x+2}\right| + 1$$

$$\left|\frac{2x+3}{x+2}\right| + 1 + x < 0 \Leftrightarrow \left|\frac{2x+3}{x+2}\right| < -1 - x$$

$$\frac{|2x+3|}{|x+2|} < \frac{-x^2 - 1 + 2x + 3}{|x+2|}$$

$$\frac{|2x+3|}{|x+2|} < \frac{-x^2 + 2x + 2}{|x+2|}$$

$$\frac{|2x+3| + x^2 - 2x - 2}{|x+2|} < 0$$

$$\frac{x^2 - 2x - 1 + |2x+3|}{|x+2|} < 0$$

$$\frac{(1+x^2+2x)-2x}{|x+2|} = \frac{(1+x)^2 - 2x}{|x+2|}$$

$$\frac{2 \pm \sqrt{4+4}}{2} \quad -1$$

$$\frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

$$\frac{1-\sqrt{2}}{1} \quad -1 \quad \frac{1+\sqrt{2}}{1}$$

$$+ \quad | \quad - \quad | \quad + \quad | \quad -$$

$$\Rightarrow \left(-1, 1-\sqrt{2}\right) \cup \left(-1, 1+\sqrt{2}\right)$$

$$\frac{|2x+3|}{|x+2|} > 0$$

$$(-\infty, -2) \cup (-2, -1)$$

$$+ \quad | \quad - \quad | \quad +$$

$$-2 \quad -1$$

$$y' = (x-2)^3 + 18$$

$$g(x) = \sqrt{f(x)(18 - f(x))}$$

$x = -1$, $x = 3$

	-1		3	
	$-$	$+$	$-$	

$$Dg = [-1, 3]$$