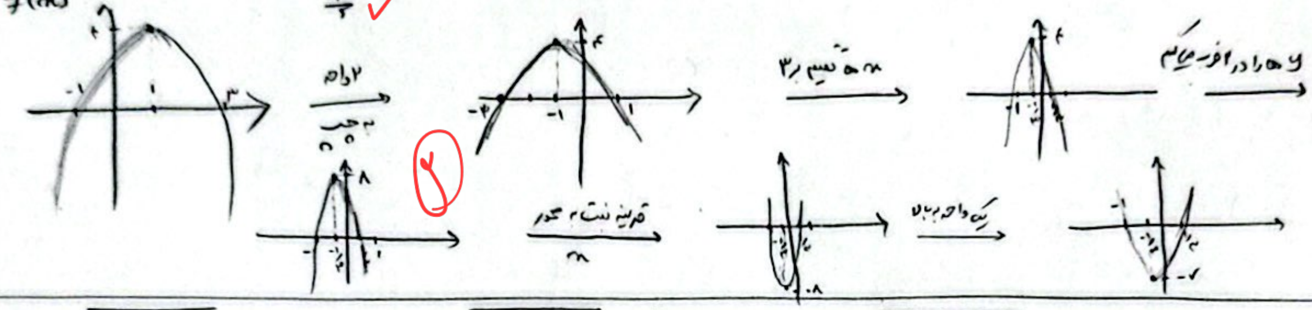
۱۶

$$-\frac{1}{3}(x-4)^2+3 \xrightarrow{y=0} -\frac{1}{3}(x-4)^2+3=0 \Rightarrow (x-4)^2=9$$

$$x-4=\pm 3 \Rightarrow \boxed{x=1,7}$$

۲ ۱

با توجه به شکل‌های رسم شده، رأس تابع جدید $(-7 , -\frac{1}{2})$ می‌باشد پس حاصل $\alpha\beta$ می‌شود $\frac{7}{2}$



۲ ۲

$$3-\sqrt{2x} \longrightarrow 3-\sqrt{2(x+1)}+3 \longrightarrow \sqrt{2(x+1)}-3+5$$

$$\longrightarrow \sqrt{2x+2}+2=\frac{7}{2} \longrightarrow \sqrt{2x+2}=\frac{3}{2} \longrightarrow 2x+2=\frac{9}{4} \longrightarrow 8x+8=9$$

$$\longrightarrow 8x=1 \longrightarrow \boxed{x=\frac{1}{8}}$$

۲ ۳

$$D_f: a<x\le 4 \longrightarrow D_{f(x-1)} \longrightarrow a<x-1\le 4 \rightarrow a+1<x\le 5$$

$$D_g: -1\le x<b \longrightarrow D_g(-2,0) \longrightarrow -1\le x+1<b \rightarrow -2\le x<b-1$$

$$(a+1 \quad 5]\cap[-2 \quad b-1) \Rightarrow (-2 \quad 5)$$

۲

$$a+1=-2 \longrightarrow a=-3 \qquad \Rightarrow a-b \longrightarrow -3-6=\boxed{-9}$$

$$b-1=5 \longrightarrow b=6$$

۴

براساس شکل $\longrightarrow -3\le x\le 5 \xrightarrow{x\times \frac{1}{2}} -\frac{3}{2}\le\frac{x}{2}\le\frac{5}{2} \longrightarrow -\frac{1}{2}\le\frac{x}{2}+1\le\frac{7}{2}$

$|2f(x)-3|\ge 0 \longrightarrow$ همواره برقرار

$$-1\le f(x)\le 2 \longrightarrow -2\le 2f(x)\le 4 \longrightarrow -5\le 2f(x)-3\le 1$$

$$\longrightarrow 0\le |2f(x)-3|\le 5 \longrightarrow 0\le\sqrt{|2f(x)-3|}\le\sqrt{5}$$

$$D_y=[-\frac{1}{2} , \frac{7}{2}] \qquad R_y=[0 , \sqrt{5}]$$

۲ ۵

$(\frac{1}{a}, 0] \in x \to y = 0 \;/\; [0, -\frac{1}{a}] \in x \to y = -\frac{x}{a}$ ۵

$(-\frac{1}{a}, -\frac{2}{a}] \in x \to -\frac{2x}{a} \rightsquigarrow x = -\frac{1}{a} \to y = \frac{-2 \times -\frac{1}{a}}{x} = \frac{1}{a^2}$ ۶

$\frac{2}{a^2} = \frac{1}{4} \to a^2 = 8 \to a = \pm\sqrt{2}$ $b=1$ $b = -\frac{2}{a} = \frac{-2}{-\sqrt{2}} = \sqrt{2}$

$f(-\frac{1}{a}) = \frac{1}{4} \to -\frac{1}{a^2}[-1] = \frac{1}{4} \to a^2 = 4 \longrightarrow$ پس $\begin{cases} a = 2 \times \\ a = -2 ✓ \end{cases}$ $b - a = \sqrt{2} - (-\sqrt{2}) = 2\sqrt{2}$

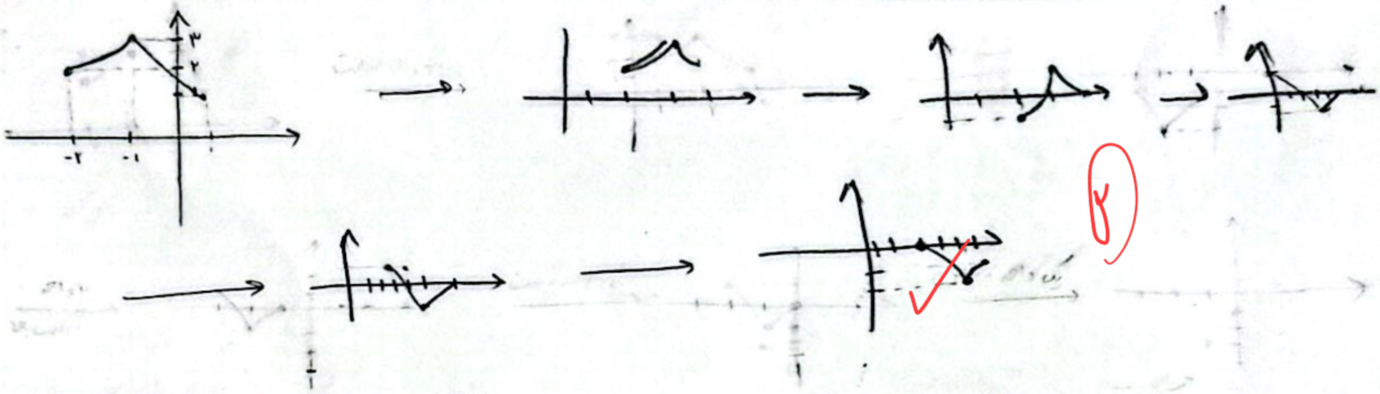
$b - a = 4$

$f(x) = \log_2^x \longrightarrow -\log_2^{(x+2)} + 3 = g(x)$

$\to g(-14) = -\log_2^{16} + 3 \to -4 + 3 = -1$

$\to g(-62) = -\log_2^{64} + 3 \to -6 + 3 = -3$ ۲ ۷

$g(-14) + g(-62) \Rightarrow -1 + (-3) = \boxed{-4}$



۲ ۸

۱

$g(x) = \left|\frac{(x-2)+1}{2(x-2)+1}\right| + 1 \longrightarrow g(x) = \frac{x-1}{2x-4+1} + 1 \longrightarrow g(x) = \left|\frac{x-1}{2x-3}\right| + 1$

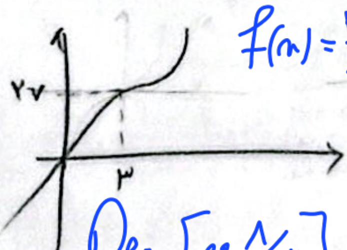
$x + 1 + \left|\frac{x-1}{2x-3}\right| < 0$ $\xrightarrow{\text{تعیین علامت}}$ ۹

1		$\frac{3}{2}$
$x + \frac{x-1}{2x-3} + 1 < 0$	$x + \frac{1-x}{2x-3} + 1 < 0$	$x + \frac{x+1}{2x-3} + 1 < 0$
$\frac{2x^2-4}{2x-3} < 0$	$\frac{2x^2-2x-2}{2x-3} < 0$	$\to \frac{2x^2-4}{2x-3} < 0$
$-\sqrt{2} \quad \sqrt{2} \quad \frac{3}{2}$: $- \mid + \mid - \mid +$	$\frac{1-\sqrt{5}}{2} \quad \frac{1+\sqrt{5}}{2} \quad \frac{3}{2}$: $- \mid + \mid - \mid +$	$(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \frac{3}{2}) \cap (\frac{3}{2}, +\infty)$
$\xleftarrow{x<1} (-\infty, -\sqrt{2}]$	$\downarrow 1 \le x \le \frac{3}{2}$ $[\frac{1+\sqrt{5}}{2}, \frac{3}{2})$	$\varnothing$

$\leftarrow x < -1$

$(-\infty, -\sqrt{2}] \cup [\cancel{\frac{1+\sqrt{5}}{2}}, \frac{3}{2}) \in \cup$

۱



$f(x) = \frac{27}{8}(x-2)^3 + 27$ چون نمودار از (۰ و ۰) گذشته پس فقط ax^3 شد

$ax^3 = f(x) \to x = 2 \to 8a = 27 \to a = \frac{27}{8}$

$\sqrt{28 f(x) - f^2(x)} = g(x) \to \sqrt{28 \times \frac{27}{8}x^3 - (\frac{27}{8}x^3)^2}$ ۱۰

$D_f = [0, 8/3]$

۳ عدد صحیح

$\longrightarrow 7 \times 27 x^3 - (\frac{27}{8})^2 x^6 \geq 0 \to x^3(7 \times 27 - (\frac{27}{8})^2 x^3) \geq 0$

$\xrightarrow{\text{تعیین علامت}}$ می‌کنیم

$0 \quad \frac{4}{3}\sqrt[3]{7}$: $- \mid + \mid -$

$D_f = [0, \frac{4}{3}\sqrt[3]{7}]$