تکلیف ۷ فاطمه نقابی ۱۶,۲۵

(۱ $y=-\frac{1}{3}x^2+\frac{2}{5}x+1 \longrightarrow (4,3)$ ent

$-\frac{1}{3}(x-4)^2+3=y \xrightarrow{y=0} -\frac{1}{3}(x-4)^2=-3 \to (x-4)^2=9$

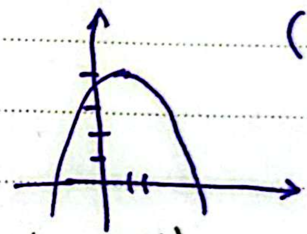
$\begin{cases} x-4=3 \to \boxed{x=7} \\ x-4=-3 \to \boxed{x=1} \end{cases}$ ۲

(۲ $y=1-2f(3x+4) \longrightarrow A=(\alpha,\beta) \to \alpha\beta=?$

$f(x) \xrightarrow[\text{انعکاس}]{} A=(1,4) \xrightarrow[\text{برعکس}]{x+4} (-1,4)$

$\xrightarrow[(\text{ضرب } \frac{1}{3})]{x\times 3} (-\frac{1}{3},4) \xrightarrow{y\times -2} (-\frac{1}{3},-8) \xrightarrow{y+1} (-\frac{1}{3},-7)$

$\alpha.\beta=-\frac{1}{3}\times -7=\boxed{\frac{7}{3}}$ ۲



α ↵ β ↵

(۳ ۱,۵

$y=3-\sqrt{2x} \xrightarrow[+1]{\text{۱ واحد چپ}} 3-\sqrt{2(x+1)}=3-\sqrt{2x+2}$

$\xrightarrow[x]{\text{قرینه نسبت به}} -(3-\sqrt{-2x-2})=\sqrt{-2x-2}-3 \xrightarrow[5+y]{\text{۵ واحد بالا}} \sqrt{2x+2}-3+5$

$\boxed{y=\sqrt{2x+2}+2}$ $\quad \sqrt{2x+2}+2=\frac{7}{2} \longrightarrow x=\frac{1}{8}$

(۴ $D_f=(a,4]$, $D_g=[-1,b]$, $y=f(x-1)+g(x+1)$, $x\in(-3,5)$ ۱,۷۵

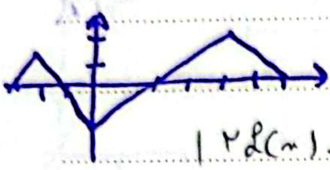
1) $f(x-1) \Rightarrow a<x-1\le 4 \to a+1<x\le 5 \to D_{f(x-1)}\in(a+1,5]$

2) $g(x+1) \to -1\le x+1\le b \to -2\le x\le b-1 \to D_{g(x+1)}\in[-2,b-1]$

$\xrightarrow{\cap} (a+1,5]\cap[-2,b-1)=(-3,5)$ $\begin{cases} a+1=-2 \to \boxed{a=-3} \\ b-1=5 \to \boxed{b=6} \end{cases}$

$a-b=-9$

۵) $y_1 = f(\frac{x}{2}+1)$; D_{y_1} , $y_2 = \sqrt{|2f(x)-3|}$, R_{y_2} } = ?



$-3 \le x \le 5 \xrightarrow{x \times \frac12} -\frac32 \le \frac{x}{2} \le \frac52 \xrightarrow{+1} -\frac12 \le \frac{x}{2}+1 \le \frac72$

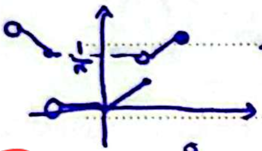
۲

$|2f(x)-3| \ge 0 \to$ همواره برقرار

$-1 \le f(x) \le 2 \xrightarrow{2f(x)} -2 \le 2f(x) \le 4 \xrightarrow{2f(x)-3} -5 \le 2f(x)-3 \le 1$

$\xrightarrow{||} 0 \le |2f(x)-3| \le 5 \xrightarrow{\sqrt{}} 0 \le \sqrt{|2f(x)-3|} \le \sqrt5$

$\to D_{y_1} = [-\frac12, \frac72]$, $R_{y_2} = [0, \sqrt5]$

۶)  $\to y = \frac{x}{a}[ax]$, $b-a=?$

$a<0$

$f(-\frac1a)=\frac14 \to -\frac{1}{a^2}[-1] = \frac14 \to a^2=4 \to a=-2$, $b=1$

$-\frac1a , \frac1a$ بازه‌بندی

۰٫۵

$(-\frac1a, 0] \in x \to y=0$ / $[0, -\frac1a] \in x \to y = -\frac{x}{a}$

$(-\frac1a, -\frac2a] \in x \to -\frac{2x}{a} \xrightarrow{\text{جایگذاری}} x = -\frac1a \to y = -\frac{2\times -\frac1a}{a} = \frac{2}{a^2}$

$\frac{2}{a^2} = \frac14 \to a = \pm\sqrt8 \xrightarrow{a<0} a=\sqrt2 , b = \frac{-2}{a} = \frac{-2}{-\sqrt2} = \sqrt2$

$\to b-a = \sqrt2-(-\sqrt2) = \boxed{2\sqrt2}$ $\boxed{b-a=3}$

۷) $f(x) = \log_2^x \xrightarrow[\text{۲ چپ}]{x+2} \log_2^{x+2} \xrightarrow[x \text{ به}]{\text{قرینه}} -\log_2^{x+2} \xrightarrow{\text{۳ واحد بالا}} -\log_2^{x+2}+3$

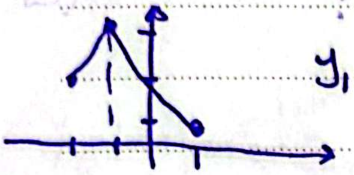
$\xrightarrow[y \text{ به}]{\text{قرینه}} -\log_2^{-x+2}+3 \to g(x) = -\log_2^{-x+2}+3 \to g(-14)=-1$, $g(-62)=-3$

$-1-3 = \boxed{-4}$

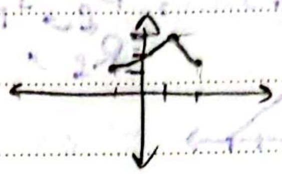
۲

$-x\ [-x]$

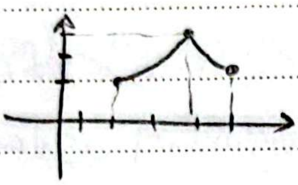
۸)



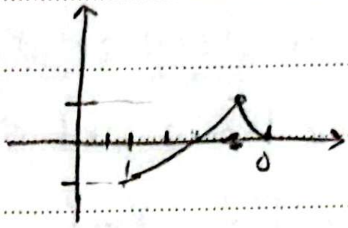
$y_1 = f\ 2 - f(3-x) \xrightarrow{\text{قرینه}}$ y به $x\times-1$



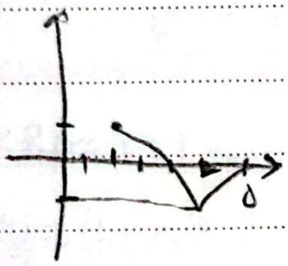
$\xrightarrow{x-3}$



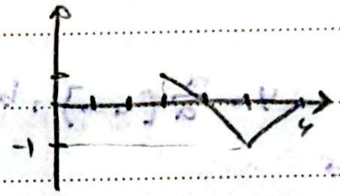
$\xrightarrow{y-2}$



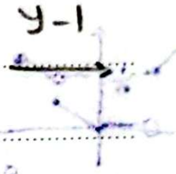
$\xrightarrow{y\times-1}$

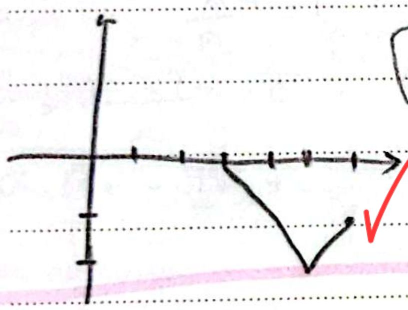


$\xrightarrow{x-1}$



$\xrightarrow{y-1}$





$\boxed{f(x-1)-1 = y}$

۲

عوللی کفر

۹)

$f(x) = \left|\frac{x+1}{2x+1}\right| \xrightarrow[\text{۲واحد راست}]{x-2} \left|\frac{(x-2)+1}{2(x-2)+1}\right| = \left|\frac{x-1}{2x-3}\right|$ ۱,۵

$\xrightarrow{y+1} \left|\frac{x-1}{2x-3}\right|+1 = g(x) \to x+g(x) < 0 \to x+\left|\frac{x-1}{2x-3}\right|+1<0$

$x<1$	$1 \le x < \frac{3}{2}$	$x \ge \frac{3}{2}$
$x+\frac{x-1}{2x-3}+1<0$	$2x+\frac{1-x}{2x-3}+1<0$	$2x-3 \to \frac{2x^2-4}{2x-3}<0$
$\frac{2x^2-3x+x-1+2x-3}{2x-3}<0$	$\frac{2x^2-3x+1-x+2x-3}{2x-3}<0$	$-\sqrt{2}\ \ \sqrt{2}\ \ \frac{3}{2}$: $-\ +\ -\ +$ $\xrightarrow{\cap}\ x\ge\frac{3}{2}$
$\frac{2x^2-4}{2x-3}<0 \to$ $-\sqrt{2}\ \ \sqrt{2}\ \ \frac{3}{2}$: $-\ +\ -\ +$	$\frac{2x^2-2x-2}{2x-3}<0 \to$ $\frac{1-\sqrt{5}}{2}\ \ \frac{1+\sqrt{5}}{2}\ \ \frac{3}{2}$: $-\ +\ -\ +$	$\varnothing \xleftarrow{\cap}$
$\xrightarrow[x<1]{\cap} (-\infty,-\sqrt{2}]$	$1<x<\frac{3}{2}\ \cap$ $[\frac{1+\sqrt{5}}{2},\frac{3}{2})$	

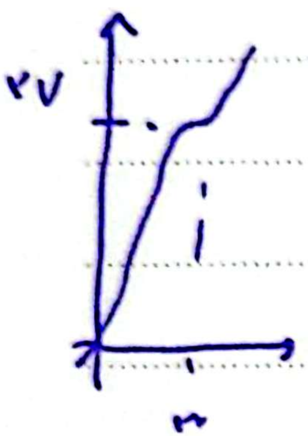
$\xrightarrow{\cup} \boxed{(-\infty,-\sqrt{2}]\cup[\frac{1+\sqrt{5}}{2},\cancel{\frac{3}{2}})}$

$x<-1$

$$f(x) = \frac{27}{8}(x-2)^3 + 27$$

$D_f = [0, 1/3]$ ← ۳ عدد صحیح

(۱۰۸



خطی از (۰,۰) گذشته ← ۷

$ax^3 = f(x) \rightarrow x = 2 \rightarrow 8a = 27 \rightarrow a = \frac{27}{8}$ ①

$f(x) = \frac{27}{8}x^3 \rightarrow \sqrt{28 f(x) - f^2(x)} = g(x) \rightarrow$

$g(x) = \sqrt{28 \times \frac{27}{8}x^3 - \left(\frac{27}{8}x^3\right)^2} \rightarrow 7 \times 28\, x^3 - \left(\frac{27}{8}\right)^2 x^6 \geq 0$

$\rightarrow x^3\left(7 \times 27 - \left(\frac{27}{8}\right)^2 x^3\right) \geq 0 \rightarrow$

	0		$\frac{4}{3}\sqrt[3]{7}$	
$-$		$+$		$-$

$\rightarrow \boxed{D_g = \left[0, \frac{4}{3}\sqrt[3]{7}\right]}$