

19.5

$$D_{g \circ f} = \{x \in D_f : f(x) \in D_g\} \quad f(x) = \sqrt{\log_p^{(x-1)}} \rightarrow x-1 > 0 \rightarrow x > 1 \rightarrow \log_p^{(x-1)} > 0$$

$$\rightarrow x-1 \geq 1 \rightarrow x \geq 2 \rightarrow D_f = [2, +\infty)$$

$$g(x) = \sqrt{-x^2 + 4x - 4} \rightarrow x^2 + 4x - 4 \geq 0 \rightarrow x^2 - 4x + 4 \leq 0 \rightarrow (x-2)^2 \leq 0 \rightarrow x=2$$

$$f(x) \in D_g \rightarrow \sqrt{\log_p^{(x-1)}} = 2 \rightarrow x-1 = 16 \rightarrow x=17$$

$$D_{g \circ f} = \{17\}$$

$$f \circ g(x) = 2(x^2 - 4x + 1) - 1 = 2x^2 - 8x + 1$$

$$g \circ f(x) = (2x-1)^2 - 2(2x-1) + 1 = 4x^2 - 4x + 1$$

$$g \circ f(x) = f \circ g(x) \rightarrow 2x^2 - 8x + 1 = 4x^2 - 4x + 1$$

$$\rightarrow 2x^2 - 4x + 0 = 0 \rightarrow x^2 - 2x = 0 \rightarrow x(x-2) = 0 \rightarrow x=0 \text{ or } x=2$$

$$x^2 + 4x - 4 = 0 \rightarrow x = \frac{-4 \pm \sqrt{16 + 16}}{2} = \frac{-4 \pm \sqrt{32}}{2} = \frac{-4 \pm 4\sqrt{2}}{2} = -2 \pm 2\sqrt{2}$$

$$f(x) = 2x \rightarrow 2x = \pm 1 \rightarrow x = \pm \frac{1}{2}$$

$$g \circ f(x) = 2\left(\frac{\pm 1}{2}\right)^2 + 1 = \frac{2}{4} \pm 1 + 1 = \frac{1}{2} \pm 1 + 1 \rightarrow g(x) = \frac{3}{2}x^2 + 1$$

$$\rightarrow g(x-1) = \frac{3}{2}(x-1)^2 + 1$$

کمترین مقدار تابع متعلق
به زمانی است که
x-1=0

$$g(0) = 1$$

$$3x^2 - 6x - 1 = 3g - 4 \rightarrow \frac{3x^2 - 6x + 3}{3} = x^2 - 2x + 1 = g(x)$$

$$g \circ f(x) = (3x-4)^2 - 2(3x-4) + 1 = 9x^2 - 24x + 16$$

$$f - g = \frac{0-0}{0-0} = -1 \rightarrow f - g = -x + 1$$

$$\begin{cases} -2f(x) + 2g(x) = 2x - 10 \\ 3f(x) - 2g(x) = 14 \end{cases} \Rightarrow f(x) = 2x + 5 \rightarrow g(x) = 3x - 1$$

$$\begin{cases} f(x) - g(x) = x + 6 \\ 3f(x) - 2g(x) = 14 \end{cases}$$

$$f \circ g(x) = 2(3x-1) + 5 = 6x + 3$$

$$\rightarrow y = \frac{2}{3}x - 1$$

$$\rightarrow \frac{2}{3}x - 1 = 6x + 3 \rightarrow a = -\frac{1}{3}$$

$$f \circ g = g(\log_2^2) = 2^{(\log_2^2)} = 2^1 = 2 \quad g \circ f = \log_2^2 = 2 \log_2^1 = 2 \cdot 1 = 2$$

$$f \circ g(x) \leq g \circ f(x) \rightarrow 2^x \leq 2x \rightarrow 2^x - 2x \leq 0 \rightarrow x(x-2) \leq 0$$

$$x \in \left[\frac{0}{1}, \frac{2}{1} \right] \rightarrow [0, 2] \rightarrow \text{مجموعه جواب}$$

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$$\frac{2^x - 2}{x} = 2 - \frac{2}{x} = \frac{1}{2} \rightarrow f\left(\frac{2^x - 2}{x}\right) = \left(2 - \frac{2}{x}\right)^2 + \left(2 - \frac{2}{x}\right) \rightarrow f\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{1}{2}$$

$$\rightarrow f(x) = 2^x + x$$

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$$f \circ g(x) = \log_2(1-x-x^2) \quad D_{f \circ g}: 1-x-x^2 > 0 \rightarrow x(1-x) > 0 \rightarrow -\frac{1}{4} < x < \frac{1}{4}$$

$$D_{f \circ g} = [0, 1] = A \quad R_g: \frac{-b}{2a} = x_{\max} = \frac{1}{2} \rightarrow y_{\max} = 1 \text{ و } x < 0 \rightarrow R_g = (-\infty, \frac{1}{2}]$$

$$D_f: x > 0 \rightarrow D_f = (0, 1] \xrightarrow{\log_2} R_{f \circ g} = (0, \frac{1}{2}] = B$$

$$A \cap B = (0, \frac{1}{2}] \cap [0, 1] = (0, \frac{1}{2}] \rightarrow \text{مجموعه جواب}$$

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$$f(f(x)) = 1 - 3f^2(x) \rightarrow f(x) = 1 - 3x^2$$

$$h(g(x)) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 4x - 2$$

$$h(g(x)) = f(x) \rightarrow x^3 - 3x^2 + 4x - 2 = 1 - 3x^2 \rightarrow x^3 + 4x - 3 = 0$$

۳ معادله درجه ۳ و صوری است پس فقط یک جواب حقیقی دارد و در نتیجه اریب دارد.

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$$f(x) = x - \frac{x}{x+1} = \frac{x^2}{x+1} \rightarrow x^2 - 3x - 4 = 0 \rightarrow (x-4)(x+1) = 0$$

$$x_1 = 4 \rightarrow g \circ f(x) = 4a + 1 = 1 \rightarrow a = 0$$

$$x_2 = -1 \rightarrow g \circ f(x) = -a - 2 = 1 \rightarrow a = -3$$

$$\Rightarrow a_1 = 0 \text{ و } a_2 = -3$$

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