Subject:
Year. Month. Date.

مریم توکلی دوازدهم A دفتر تکلیف ۸

سوال ۱) $f(x)=\sqrt{\log_2^{(x-1)}}$ $g(x)=\sqrt{-x^2+6x-8}$

$D_{gof(x)}=?$

$gof \rightarrow f(x)=t \rightarrow \sqrt{-t^2+6t-8}=\sqrt{-(t-3)^2}$ تنها حالتی که می‌توانیم جواب داشته باشیم ←

$t-3=0 \Rightarrow \underline{t=3}$

$t=3 \rightarrow f(x)=3 \rightarrow \sqrt{\log_2^{(x-1)}}=3 \rightarrow \log_2^{x-1}=9$

$\Rightarrow x-1=16 \rightarrow \boxed{x=17}$

$\boxed{D_{gof(x)}=\{17\}}$

سوال ۲) $f(x)=3x-5$ $g(x)=x^2-3x+8$ $fog(x)=gof(x)$

$\alpha^2+\beta^2=?$

$fog(x)=3(x^2-3x+8)-5 \rightarrow 3x^2-9x+19$

$gof(x)=(3x-5)^2-3(3x-5)+8=9x^2-39x+48$

$9x^2-39x+48=3x^2-9x+19 \Rightarrow 3x^2-30x+29=0$

$\alpha^2+\beta^2=S^2-2P \rightarrow S=\frac{30}{3}=10 \rightarrow S^2-2P=100-58=\boxed{42}$

$P=\frac{29}{3}$

سوال ۳) $gof(x)=5x^2+11$ $f(x)=2x$ $\min g(x-7)=?$

$f(x)=2x \rightarrow 2x=t \rightarrow x=\frac{t}{2} \rightarrow 5\left(\frac{t}{2}\right)^2+11=g(t)$

$\Rightarrow g(x)=\frac{5x^2}{4}+11 \rightarrow g(x-7)=\frac{5(x-7)^2}{4}+11$

$\Rightarrow g(x-7)=\frac{5x^2}{4}-\frac{70x}{4}+\frac{245}{4}+11$

ادامه‌ی حل در عکس بعد

Subject:
Year. Month. Date.

ضریب x^2 مثبت است. پس تابع دارای min است،

$$\min \begin{cases} -\frac{b}{2a} = 1 \\ y_{min} = \boxed{11} \end{cases}$$

سوال ۴) $f(x) = 2x - 4$ $fog(x) = 2x^2 - 4x - 1$ $gof(x) = ?$

$$\rightarrow fog(x) = 2g - 4 = 2x^2 - 4x - 1 \Rightarrow 2g = 2x^2 - 4x + 2$$

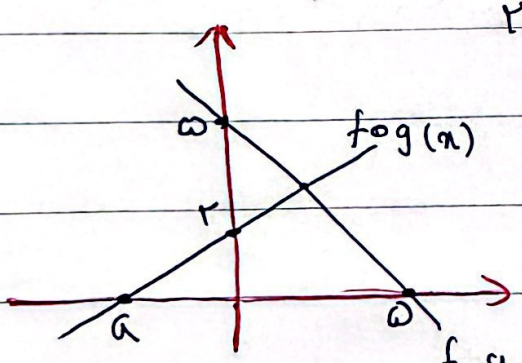
$$\Rightarrow g(x) = x^2 - 2x + 1 \qquad gof(x) = (2x-4)^2 - 2(2x-4) + 1$$

$$\Rightarrow \boxed{4x^2 - 20x + 25}$$

سوال ۵)

$$2g(x) = 3f(x) - 14$$

$$\searrow \quad g(x) = \frac{3}{2}f(x) - 7 \rightarrow$$ هر دو تابع خطی هستند



نمودار $f-g$ به صورت روبرو است

$$f-g(x) = -x+5 \Rightarrow f(x) - \frac{3}{2}f(x) + 7 = -\frac{1}{2}f(x) + 7 = f-g(x)$$

$$\Rightarrow -\frac{1}{2}f(x) + 7 = -x + 5 \Rightarrow f(x) = 2x + 4$$

$$g(x) = \frac{3}{2}(2x+4) - 7 \Rightarrow g(x) = 3x - 1$$

$$\rightarrow fog(x) = 2(3x-1) + 4 = 6x + 2$$

$$\Rightarrow fog(x) = 0 \Rightarrow 6x + 2 = 0 \Rightarrow x = -\frac{1}{3} \Rightarrow \boxed{a = -\frac{1}{3}}$$

سوال ۶) $f(x) = 9^x$ $g(x) = \log_3^x$ $fog(x) \leq gof(x)$

$$fog(x) = 9^{\log_2^x} = x^{\log_2^9} = x^2 \qquad x^2 \leq 2x \rightarrow x^2 - 2x \leq 0$$

$$gof(x) = \log_3^{9^x} = \log_3^{3^{2x}} = 2x \qquad \rightarrow x(x-2) \leq 0$$

$$\begin{array}{ccccc} & + & 0 & - & 2 & + \\ \end{array} \rightarrow x \in [0, 2]$$

۲ عدد صحیح

$\log(x)$ به ازای $x \neq 0$ باید باشد. چون شرطش $x > 0$ است

سوال ۷) $f\left(\frac{x^2-2}{x}\right) = x^2+x-3-\frac{2}{x}+\frac{4}{x^2}$ $f(x)=?$

$$f\left(x-\frac{2}{x}\right) = \frac{x^2+\frac{4}{x^2}-4}{\left(x-\frac{2}{x}\right)^2} + x - \frac{2}{x} \Rightarrow f(t) = t^2+t \Rightarrow \boxed{f(x)=x^2+x}$$

سوال ۸) $f(x)=\log_2^{x}$ $g(x)=8x-x^2$ $A\cap B$ ← تابع صعودی؟

$$fog(x) = \log_2^{-x^2+8x} \Rightarrow Dfog(x) = \underbrace{Dg}_{R}, \underbrace{g(x)\in Df}_{\downarrow} = \underbrace{(0,8)}_{A}$$

$$-x^2+8x>0$$

	0		8	
	−	+	−	

$$R_{fog(x)} = \underbrace{[1}_{B} \Rightarrow A\cap B = (0,8) \rightarrow \boxed{\text{تابع صعودی}}$$

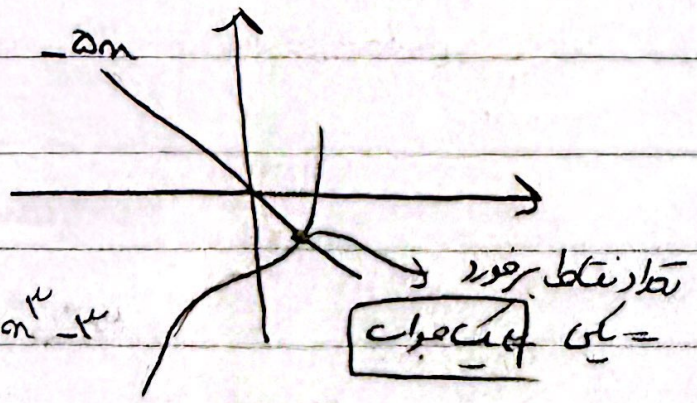
سوال ۹) $fof(x) = 1-2f^2(x)$ $g(x)=x-1$ $h(x)=x^2+2x+1$

$$\underbrace{h(g(x))}_{\downarrow} = f(x)$$

$$(x-1)^2+2(x-1)+1 = x^2-2x^2+3x-2$$

$$f(f(x)) = 1-2f^2(x) \Rightarrow f(x) = 1-2x^2$$

$$x^2-2x^2+3x-2 = 1-2x^2 \Rightarrow x^2+3x-3=0 \Rightarrow x^2-2=-3x$$



Subject:
Year. Month. Date.

سوال ۱۰۱)

$f(x)=x-\frac{4}{x}$ $\quad gof(x)=ax^2+2x$

$g(2)=8 \quad a=?$

$g(x-\frac{4}{x})=ax^2+2x$

$x-\frac{4}{x}=2 \rightarrow x^2-4-2x=0 \rightarrow (x-4)(x+1)=0$

$x=4$, $x=-1$

$x=-1 \rightarrow -a-2=8 \Rightarrow \boxed{a=-10}$

$x=4 \rightarrow 4(4)a+8=8 \Rightarrow a=0$ ق ق ق $\rightarrow a\neq 0$