آیلا فیروزنیا　　19

۱. $f(x)=\sqrt{\log_2^{(x-1)}}$　$D_f: x-1>0 \to x>1$
$\log_2^{(x-1)} \geq 0 \to x-1\geq 1 \to x\geq 2$) $D_f=[2,+\infty)$

$g(x)=\sqrt{-(x^2-4x+4)}$　$D_g: -(x-2)^2\geq 0 \to x=2$

$D_{gof} \Rightarrow D_f: [2,+\infty)$

$f(x)\in D_g \to \sqrt{\log_2^{(x-1)}}=2 \to \log_2^{(x-1)}=4 \to x-1=16 \to x=17 \to D_{gof}=\{17\}$　✓ 2

۲. $f(x)=2(x^2-3x+8)-5=2x^2-6x+11$

$gof=(2x-5)^2-3(2x-5)+8=4x^2-20x+25-6x+15+8=4x^2-26x+48$

$2x^2-6x+11=4x^2-26x+48 \Rightarrow 2x^2-20x+37=0$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\frac{-b}{a}=\frac{20}{2}=10$　$\frac{c}{a}=\frac{37}{2}\times 2=37$

$\to 10^2-37=100-37=63$　✓ 2

۳. $g(f(x))=g(2x)=5x^2+11$

$g(x)=5(\frac{x}{2})^2+11 \to g(x)=\frac{5}{4}x^2+11 \to g(x-7)=\frac{5}{4}(x-7)^2+11$ (مربع مثبت یا صفر)

$g(x-7)_{min}=\frac{5}{4}(0)^2+11=11$　✓ 2

آیلا هرودنیا

۴. $3(g(x)) - 4 = 3x^2 - 6x - 1 \to 3g(x) = 3x^2 - 6x + 3$

$g(x) = x^2 - 2x + 1$

$gof(x) = (3x - 4)^2 - 2(3x - 2) + 1 = 9x^2 + 16 - 24x - 6x + 8 + 1$

$= 9x^2 - 30x + 25 =$ ✓ ۲

Date No

نیکلا صورتیا

۵. هر دو طرف $\div 2$ ـ $2g(x) = 3f(x) - 14 \rightarrow g(x) = \frac{3}{2}f(x) - 7$

$f - g(x) = -x + 5 \rightarrow f(x) - \frac{3}{2}f(x) + 7 = -\frac{1}{2}f(x) + 7 = f - g(x)$

$\rightarrow -\frac{1}{2}f(x) + 7 = -x + 5$

$f(x) = 2x + 4$

$g(x) = \frac{3}{2}(2x + 4) - 7 \rightarrow g(x) = 3x - 1 \rightarrow fog(x) = 2(3x - 1) + 4 = 6x + 2$

۲

$fog(x) = 0 \rightarrow 6x + 2 = 0 \rightarrow 6x = -2 \rightarrow x = -\frac{1}{3}$ $\boxed{x = -\frac{1}{3}}$ ✓

۶- $f\circ g(x) = 9^{\log_3 x} = 3^{2\log_3 x} = (3^{\log_3 x})^2 = x^2 \rightarrow f\circ g(x) = x^2$

($9^{\log_3 x} = x^2$)

$g\circ f = g(9^x) = \log_3 9^x = x\log_3 9 = 2x$

$x^2 \leq 2x \rightarrow x^2 - 2x \leq 0 \rightarrow x(x-2) \leq 0$

	0		2	
+	0	−	0	+

(۲)

ولی صفر در مجموعه نمی‌تونه باشه پس ← ۱، ۲ (۲ عدد صحیح) ✓

۷. $t = \frac{x^2-2}{x} = x - \frac{2}{x}$

$x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2} = \left(x^2 - 4 + \frac{4}{x^2}\right) + \left(x - \frac{2}{x}\right) \sim t^2 + t$

پس $f(x) = x^2 + x$ ✓ (۴)

۸. $f(x) = \log_2 x$

$g(x) = -x^2 + 8x$

(۱)

$f\circ g(x) = \log_2^{-x^2+8x} \rightarrow \overset{A}{D_{f\circ g}} = \underset{\mathbb{R}}{D_g},\ g(x) \in D_f = (0,8)$

$-x^2 + 8x > 0 \rightarrow -x(x-8) > 0$

	0		8	
−	0	+	0	−

B

$R_{f\circ g} = \mathbb{R} \rightarrow A \cap B = (0,8)$

۷ عدد صحیح

$B = (-\infty, 4]$

$A \cap B = (0, 4]$ → ۴ عدد صحیح

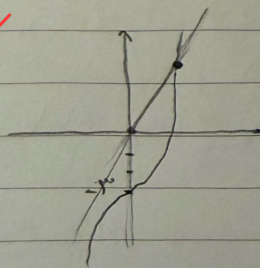
آحلا هتروذنا

۹. $f\circ f(x) = 1 - 3f^2(x) , f(f(x)) = 1 - 3f^2(x) \rightarrow f(x) = 1 - 3x^2$

$g(x) = x - 1$
$h(x) = x^3 + 2x + 1$
$\Big\} h(g(x)) = h(x-1) = (x-1)^3 + 2(x-1) + 1 = (x-1)^3 + 2x - 1$

$h(g(x)) = f(x) \rightarrow x^3 - 3x^2 + 3x - 1 + 2x - 1 = 1 - 3x^2$

$x^3 + 5x - 3 = 0 \rightarrow x^3 - 3 = 5x$



۲

— یک برخورد پس یک جواب —

۱۰. $f(x) = 3 \rightarrow x - \frac{4}{x} = 3 \rightarrow x^2 - 4 = 3x \rightarrow x^2 - 3x - 4 = 0$

$x = \frac{3 \pm \sqrt{9 + 16}}{2} \rightarrow 4 , -1$

۲

if $x = -1 \rightarrow g(f(-1)) = g(3) = a(-1)^3 + 2(-1) = -a - 2 \rightarrow -a - 2 = 8 \rightarrow a = -10$

$g(f(4)) = g(3) = a(4^3) + 2(4) = 8 \rightarrow 64a + 8 = 8 \rightarrow 64a = 0 \rightarrow a = 0$