نام و نام خانوادگی: ... ۱۹.۵ ... پاسخنامه تشریحی شماره ۸

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$$g(f(x)) \longrightarrow \{x \in D_f \,/\, f(x) \in D_g\}$$

$D_{f(x)} = \sqrt{\log_2^{x-1}} \longrightarrow x-1>0 \to x>1$; $\log_2^{x-1} \geq 0 \to x-1 \geq 1 \to x \geq 2$ $\xrightarrow{\text{اشتراک}} [2, +\infty)$

$\sqrt{\log_2^{x-1}} \longrightarrow \log_2^{x-1} = 4 \to x-1 = 16 \to \boxed{x = 17}$

$D_{g(x)} = \sqrt{-x^2+4x-4} \to \sqrt{-(x-2)^2} \longrightarrow x-2=0 \to x=2$

$D_g = \{2\}$

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$f(g(x)) = g(f(x))$

$f(x^2-3x+8) = g(2x-5)$

$2(x^2-3x+8) - 5 = (2x-5)^2 - 3(2x-5) + 8$

$\cancel{2x^2} - 6x + 16 - 5 = \cancel{4x^2} + 25 - 20x - 6x + 15 + 8$

$2x^2 - 20x = 25+15+8-11 \Longrightarrow 2x^2 - 20x + 37 = 0 \longrightarrow \alpha^2+\beta^2 \to S^2 - 2P \to 100 - 37 = 63$

$S = 10 \quad P = \frac{37}{2}$

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$g(\underbrace{f(x)}_{2x}) = 5x^2+11 \longrightarrow g(2x) = \underbrace{5x^2+11} \xrightarrow{\text{طول رأس}} \frac{-b}{2a} = 0$

$1 \xrightarrow[\text{عرض}]{\text{جایگذاری}} \boxed{11}$ کمترین مقدار

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$f(x) = 3x-4$

$f(g(x)) = 3x^2-6x-1 \longrightarrow 3g(x) - 4 = 3x^2-6x-1$

$\longrightarrow 3g(x) = 3x^2-6x+3 \longrightarrow g(x) = x^2-2x+1$

$g(f(x)) \longrightarrow g(3x-4) \longrightarrow (3x-4)^2 - 2(3x-4) + 1$

$9x^2+16-24x-6x+8+1 \longrightarrow 9x^2-30x+25 = g(f(x))$

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۵

$f - g = -x+5 \longrightarrow$ $3f(x) - 3g(x) = -3x+15$

$2g(x) - 3f(x) = -14$; $-3f(x) + 2g(x) = -14$

$f(x) = 2x+4$

$-g(x) = -3x+1 \longrightarrow g(x) = 3x-1$

$fog(x) = 2(3x-1) + 4 = 6x+2 \longrightarrow 6x+2 = 0 \longrightarrow x = a = \frac{-2}{6} = -\frac{1}{3}$

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$f(g(x)) \leq g(f(x)) \longrightarrow f(\log_3^x) \leq g(9^x)$

$9^{\log_3^x} \leq \log_3^{9^x} \longrightarrow x^{\log_3^9} \leq x \log_3^9 \longrightarrow x^2 \leq 2x$

$x^2 - 2x \leq 0 \longrightarrow x(x-2) \leq 0$ $\quad$ $+ \; 0 \; - \; 2 \; +$ $\longrightarrow [0, 2]$ ۶

$\longrightarrow 0, 1, 2 \Rightarrow$ (۳ عضو صحیح) ۲

$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2} \longrightarrow f\left(x - \frac{2}{x}\right) = x^2 + \frac{4}{x^2} - 4 + x - \frac{2}{x} \Rightarrow$

$f(\underbrace{x - \tfrac{2}{x}}_{t}) = \left(x - \frac{2}{x}\right)^2 + x - \frac{2}{x} \rightarrow f(t) = t^2 + t \xRightarrow{x=t} f(x) = x^2 + x$ ۷

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$f(g(x)) \rightarrow f(8x - x^2) \longrightarrow \log_2^{8x-x^2}$ ۱/۵

دامنه $\rightarrow 8x - x^2 > 0 \longrightarrow 0 > x^2 - 8x \longrightarrow 0 > x(x-8)$ $\quad$ $+ \; 0 \; - \; 8 \; +$

$(0, 8) = A$

$D_{fog} = (0 \quad 8)$ ۸

$R\, fog(x) \rightarrow R_f \rightarrow x \in D_{fog} \longrightarrow \log_2^{8x-x^2} = fog$ $\quad (-\infty, 4]$ $\quad$ ۳ عضو ۴ $\quad$ ۳,۲,۱

$R(8x - x^2) \longrightarrow \frac{-b}{2a} = 4 \longrightarrow 32 - 16 = 16 \rightarrow R\, g = (0, 16)$

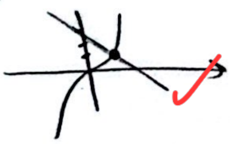
$\log_2^{x'} \longrightarrow$ $\qquad \longrightarrow R_{fog} = (-\infty \; 4] \cap (0, 8) = (0, 4]$

$f(f(x)) = 1 - 3f^2(x) \longrightarrow f(t) = 1 - 3t^2 \longrightarrow f(x) = 1 - 3x^2$

$h(g(x)) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 3x - 1 + 2x - 2 + 1$

$h(g(x)) = x^3 - 3x^2 + 5x - 2 = 1 - 3x^2 \longrightarrow x^3 + 5x - 3 = 0$ ۹

$x^3 = -5x + 3$



$\longrightarrow$ یه نقطه برخورد دارند پس معادله یک جواب دارد.

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$g(f(x)) = ax^3 + 2x$ $\qquad (x - \frac{4}{x} = 3) \times x$

$g\left(x - \frac{4}{x}\right) = ax^3 + 2x$ $\qquad \hookrightarrow x^2 - 4 = 3x \rightarrow x^2 - 3x - 4 = 0$

$\longrightarrow (x+1)(x-4) = 0 \rightarrow x = 4, \; x = -1$

$\xrightarrow{x=4} g(3) = 64a + 8 = 8 \rightarrow 64a = 0 \rightarrow a = 0$ ۱۰

$\xrightarrow{x=-1} g(3) = -a - 2 = 8 \longrightarrow -a = 10 \rightarrow a = -10$ ۲