۱)

$gof(x)=\sqrt{-(\sqrt{\log_2^{x-1}})^2+4(\sqrt{\log_2^{x-1}})-4} \xrightarrow{\sqrt{\log_2^{x-1}}=t} -t^2+4t-4\geq 0 \to$

$-(t^2-4t+4)\geq 0 \to -\underbrace{(t-2)^2}_{\text{همواره + یا صفر}}\geq 0 \to t-2=0 \to t=2 \to \log_2^{x-1}=4 \to x-1=16 \to$

(1) $x=17$

(2) $x-1>0 \to x>1$

(3) $\log_2^{x-1}\geq 0 \to x-1\geq 0 \to x\geq 1 \xrightarrow{x=1\ \times} x>1 \longrightarrow D_{gof}=\{17\}$

۲)

$fog(x)=2(x^2-3x+8)-5=2x^2-6x+11 \to fog(x)=gof(x) \longrightarrow$

$gof(x)=\underbrace{(2x-5)^2}_{4x^2-20x+25}-\underbrace{3(2x-5)}_{6x-15}+8=4x^2-26x+48$

$2x^2-6x+11=4x^2-26x+48 \longrightarrow$

$2x^2-20x+37=0 \longrightarrow$

$x=\frac{20\pm\sqrt{400-296}}{4}=\frac{20\pm\sqrt{104}}{4}=$

$\to 5+\frac{\sqrt{26}}{2} \xrightarrow{\text{به توان ۲}} 25+\frac{26}{4}+5\sqrt{26}=\alpha$

$\to 5-\frac{\sqrt{26}}{2} \xrightarrow{\text{به توان ۲}} 25+\frac{26}{4}-5\sqrt{26}=\beta$

$\alpha+\beta=50+13=\boxed{63}$

۳)

$gof(x)=a(2x)^2+11=5x^2+11 \to a(4x^2)=5x^2 \to 4a=5 \to a=\frac{5}{4}$

$g(x)=\frac{5}{4}x^2+11 \xrightarrow{x=7} g(x-7)=g(0)=\boxed{11=\min g(x-7)}$

۴)

$fog(x)=3(g(x))-4=3x^2-6x-1 \to 3(g(x))=3x^2-6x+3 \longrightarrow$

$g(x)=x^2-2x+1 \longrightarrow gof(x)=\underbrace{(3x-4)^2}_{9x^2-24x+16}-\underbrace{2(3x-4)}_{-6x+8}+1=\boxed{9x^2-30x+25}$

۵)

$2g(x)=3f(x)-14 \to g(x)=\frac{3}{2}f(x)-7 \longrightarrow$ هر دو تابع خطی‌اند

$f-g(x)=-x+5 \to f(x)-\frac{3}{2}f(x)+7=-\frac{1}{2}f(x)+7=f-g(x) \longrightarrow$

$-\frac{1}{2}f(x)+7=-x+5 \to f(x)=2x+4$

$\to g(x)=\frac{3}{2}(2x+4)-7 \to g(x)=3x-1 \to fog(x)=2(3x-1)+4=6x+2 \to$

$fog(x)=0 \to 6x+2=0 \to x=-\frac{1}{3} \to a=-\frac{1}{3}$

۶)

$f(x)=9^x$

$g(x)=\log_3^x$

$fog(x)=9^{\log_3^x}=x^{\log_3^9}=x^2$

$gof(x)=\log_3^{9^x}=\log_3^{3^{2x}}=2x$

$fog(x)\leq gof(x) \longrightarrow$

$x^2\leq 2x \to x^2-2x\leq 0 \longrightarrow$

$x(x-2)\leq 0$

$0 \qquad 2$

$+ \quad 0 \quad - \quad 2 \quad + \to x\in[0,2] \to$ ۳ عدد صحیح

(۷

$$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2} \longrightarrow \left(x^2 - 4 + \frac{4}{x^2}\right) + x - \frac{2}{x} = \left(x-\frac{2}{x}\right)^2 + \left(x - \frac{2}{x}\right) \rightarrow$$

$$\frac{x^2-2}{x} = x - \frac{2}{x} \xrightarrow{\text{به توان ۲}} \left(x-\frac{2}{x}\right)^2 = x^2 - 4 + \frac{4}{x^2}$$

$$f\left(x-\frac{2}{x}\right) = \left(x-\frac{2}{x}\right)^2 + \left(x-\frac{2}{x}\right)$$

$$\boxed{f(x) = x^2 + x}$$

(۸

$$f(x) = \log_2^x \qquad g(x) = -x^2 + 8x$$

$$fog(x) = \log_2^{-x^2+8x} \rightarrow D_{fog} = \underbrace{D_g}_{\mathbb{R}} , \underbrace{g(x) \in D_f}_{-x^2+8x>0} = (0,8) = A$$

$$-x(x-8) > 0 \quad 0 \swarrow \searrow 8$$

$$\begin{array}{ccccc} & 0 & & 8 & \\ - & | & + & | & - \end{array}$$

$$R_{fog} = \overset{B}{\mathbb{R}} \longrightarrow A \cap B = (0,8) \longrightarrow \boxed{\text{۷ عضو صحیح}}$$

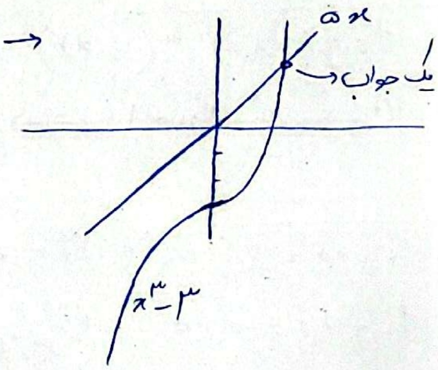
(۹

$$fof(x) = 1 - 3f^2(x) \rightarrow f(f(x)) = 1 - 3f^2(x) \rightarrow f(x) = 1 - 3x^2$$

$$\left.\begin{array}{l} g(x) = x-1 \\ h(x) = x^3 + 2x + 1 \end{array}\right\} \rightarrow h(g(x)) = (x-1)^3 + 2(x-1) + 1 = (x-1)^3 + 2x - 1$$

$$h(g(x)) = f(x) \rightarrow x^3 - 3x^2 + 3x - 1 + 2x - 1 = 1 - 3x^2 \rightarrow$$

$$h(g(x)) = f(x) \qquad x^3 + 5x - 3 = 0 \rightarrow x^3 - 3 = 5x \rightarrow$$



(۱۰

$$\left.\begin{array}{l} f(x) = x - \frac{4}{x} \\ gof(x) = ax^3 + 2x \\ g(3) = 8 \end{array}\right\} \rightarrow g\left(x - \frac{4}{x}\right) = ax^3 + 2x$$

$$x - \frac{4}{x} = 3 \rightarrow x^2 - 3x - 4 = 0 \rightarrow (x-4)(x+1) = 0 \longrightarrow$$

$$\begin{cases} x = -1 \rightarrow -a - 2 = 8 \rightarrow \boxed{a = -10} \checkmark \\ x = 4 \rightarrow 64a + 8 = 8 \rightarrow a = 0 \ \times \end{cases}$$