۱ـ

$f(x)=\sqrt{\log_2^{(x-1)}}$ $\quad g(x)=\sqrt{-x^2+4x-4}$ $\quad D_{gof}(x)=?$

$D_{gof}(x)=\{x\in D_f : f(x)\in D_g\}$

$\log_2^{(x-1)}\geq 0 \rightarrow x-1\geq 1 \rightarrow x\geq 2$

$x-1>0 \rightarrow x>1$

$D_f = x\geq 2$

$-x^2+4x-4\geq 0 \rightarrow x^2-4x+4\leq 0 \rightarrow (x-2)^2\leq 0$

	2	
+	0	+

عبارت $(x-2)^2\leq 0$ پس تنها $(x-2)^2=0$ قابل قبول است پس $D_g: x=2$

$D_{gof}:\{x\geq 2 \; ; \; \underbrace{\sqrt{\log_2^{(x-1)}}=2}\}$

$\rightarrow \log_2^{(x-1)}=4 \rightarrow \log_2^{x-1}=\log_2^{16} \rightarrow x-1=16$

$x=17$

$\Rightarrow D_{gof}=\{17\}$

۲ـ

$f(x)=2x-5 \qquad g(x)=x^2-3x+8$

$fog(x)=gof(x)$

$f(g(x))=2(x^2-3x+8)-5=2x^2-6x+16-5=2x^2-6x+11$

$g(f(x))=(2x-5)^2-3(2x-5)+8=$

$4x^2-20x+25-6x+15+8=4x^2-26x+48$

$fog(x)=gof(x)\Rightarrow 2x^2-6x+11=4x^2-26x+48$

$2x^2-20x+37=0$ $\qquad$ ریشه ها: β,α

$x^2-(10)x+\left(\frac{37}{2}\right)=0$

$\alpha^2+\beta^2=S^2-2P$

$x^2-(S)x+P=0$

$\alpha^2+\beta^2=100-2\left(\frac{37}{2}\right)=100-37=\boxed{63}$

۳-

$$g\circ f(x)=5x^2+11 \qquad f(x)=2x \qquad \min g(x-7)=?$$

$$g(f(x))=5x^2+11 \qquad f(x)=2x=t$$

$$x=\frac{t}{2}$$

$$g(t)=5\left(\frac{t}{2}\right)^2+11$$

$$g(t)=\frac{5}{4}t^2+11 \Rightarrow g(x)=\frac{5}{4}x^2+11$$

$$g(x-7)=\frac{5}{4}(x-7)^2+11=\frac{5}{4}(\underbrace{x^2-14x+49}_{\min=0})+11$$

$$x^2-14x+49 \to ext \;\Big|\; \frac{-b}{2a}=\frac{14}{2}=7 \qquad 49-98+49=0$$

$$\Rightarrow \boxed{\min g(x-7)=11}$$

۴-

$$f(x)=3x-4 \qquad \Rightarrow g\circ f(x)=?$$

$$f\circ g(x)=3x^2-6x-1$$

$$\left.\begin{array}{l} f(g(x))=3x^2-6x-1 \\ f(g(x))=3g-4 \end{array}\right| \Rightarrow 3g-4=3x^2-6x-1$$

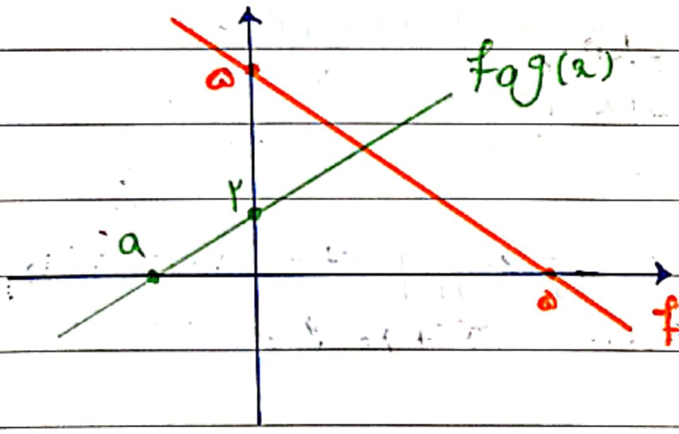
$$3g(x)=3x^2-6x+3$$

$$g(x)=x^2-2x+1=(x-1)^2$$

$$g\circ f(x)=g(f(x))=(3x-4-1)^2=(3x-5)^2 \Rightarrow$$

$$\boxed{g\circ f(x)=9x^2-30x+25}$$

۵-



$$2g(x)=3f(x)-14$$

مطابق شکل می‌توان نتیجه گرفت:

$$f(x)=ax+b$$

$$g(x)=cx+d$$

طبق نمودار $f-g$:

$$f-g(x)=-x+5$$

$$f(x)-g(x)=ax+b-cx-d=(a-c)x+b-d=-x+5 \qquad (*)$$

$$\Rightarrow a-c=-1, \quad b-d=5$$

$$2g(x)=3f(x)-14 \quad (*)$$

$$2cx+2d=3ax+3b-14$$

$$2c=3a \qquad 2d=3b-14$$

$$c=\frac{3}{2}a$$

$$(*) \Rightarrow a-\frac{3}{2}a=-\frac{1}{2}a=-1 \Rightarrow \boxed{a=2} \quad \boxed{c=3}$$

← ادامه در صفحه بعد:

$b - d = 5 \quad \times(-2)$
$3b - 2d = 14$

$$\begin{cases} -2b + 2d = -10 \\ 3b - 2d = 14 \end{cases} \quad +$$

$\underline{b = 4 \,,\; d = -1}$

$\Rightarrow f(x) = ax + b = 2x + 4$
$\Rightarrow g(x) = cx + d = 3x - 1$
$\Rightarrow f(g(x)) = 2(3x - 1) + 4 = 6x - 2 + 4 = 6x + 2$

$fog(x) = 6x + 2 \xrightarrow[\text{محور xها}]{\text{محل برخورد با}} 6x + 2 = 0 \rightarrow x = -\frac{1}{3} \Rightarrow \boxed{a = -\frac{1}{3}}$

۶-

$f(x) = 9^x \qquad g(x) = \log_3 x \qquad fog(x) \leq gof(x)$

$fog(x) = f(g(x)) = 9^{\log_3 x} = x^{\log_3 9} = x^2$
$gof(x) = g(f(x)) = \log_3 9^x = x \log_3 9 = 2x$
$\Rightarrow x^2 \leq 2x$

$x^2 - 2x \leq 0$
$x(x - 2) \leq 0$ → 0 , 2

	0		2	
$+$		$-$		$+$

$\rightarrow x \in [0, 2] \xrightarrow{x \in \mathbb{Z}}$ ۰ و ۱ و ۲

تعداد اعضای صحیح: سه عضو

۸-

$f(x) = \log_2 x \qquad g(x) = 8x - x^2$

$A \rightarrow D_{fog} \qquad B \rightarrow R_{fog} \qquad \Rightarrow A \cap B = ?!$

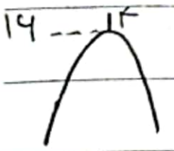
$D_{fog} : \{ x \in D_g \,;\, g(x) \in D_f \}$

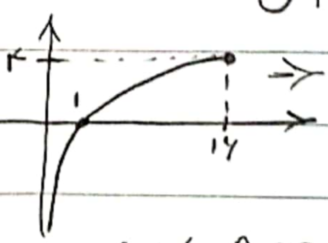
	0		8	
$+$		$-$		$+$

$D_f : x > 0 \Rightarrow x \in \mathbb{R} \,;\, -x^2 + 8x > 0$
$D_g : \mathbb{R}$
$x^2 - 8x < 0$
$x(x - 8) < 0$ → 0 , 8
$\Rightarrow D_{fog} : (0, 8) = A$

خروجی تابع g ورودی تابع f می‌شود و خروجی f برابر تابع fog می‌شود پس:

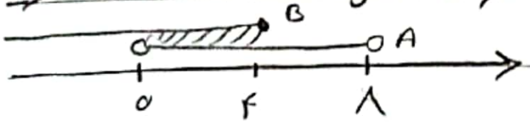
$-x^2 + 8x \rightarrow R_g = D_f = x \leq 16 \qquad f(x) = \log_2 x \;\; * \; x > 0$





$\Rightarrow R_{fog} = (-\infty, 4] = B$

$\Rightarrow A \cap B = (0, 4] \xrightarrow{x \in \mathbb{Z}} 1, 2, 3, 4$



پس تعداد اعضای صحیح $A \cap B$: ۴ عضو

$$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2} \longrightarrow f(x) \qquad -7$$

$$f\left(\frac{x^2-2}{x}\right) = \left(\frac{x^2-2}{x}\right)^2 + \frac{x^2-2}{x}$$

$$f(x) = x^2 + x$$

$$f\circ f(x) = 1 - 3f^2(x) \qquad h(g(x)) = f(x)$$ ۹-

$$g(x) = x - 1$$

$$h(x) = x^3 + 2x + 1$$

$$f(f(x)) = 1 - 3f^2(x) \longrightarrow f(t) = 1 - 3t^2 \longrightarrow f(x) = 1 - 3x^2$$

$$f(x) = t$$

$$h(g(x)) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 3x - 1 + 2x - 1$$

$$h(g(x)) = x^3 - 3x^2 + 5x - 2$$

$$\Rightarrow h(g(x)) = f(x) \longrightarrow x^3 - \cancel{3x^2} + 5x - 2 = 1 - \cancel{3x^2}$$

$$x^3 + 5x - 2 = 1 \longrightarrow x^3 + 5x - 3 = 0$$

$$3x^2 + 5 = 0 \qquad \Delta y' = 0 - 4(3)(5) = -20 \longrightarrow \Delta y' < 0$$

چون دلتای مشتق عبارت منفی است پس معادله‌ی اصلی دارای یک ریشه است

$$f(x) = x - \frac{4}{x} \qquad g\circ f(x) = ax^3 + 2x$$ ۱۰-

$$g(\underset{3}{3}) = \underset{8}{8} \Rightarrow a = ?$$

$$g(f(x)) = ax^3 + 2x \longrightarrow$$

$$f(x) = x - \frac{4}{x} = 3 \longrightarrow x^2 - 4 = 3x$$

$$x^2 - 3x - 4 = 0 \qquad (x-4)(x+1) = 0$$

$$x = 4 \qquad x = -1$$

$$g(\overset{3}{f(-1)}) = -a - 2 = 8 \longrightarrow -a = 10 \longrightarrow a = -10 \quad ✗$$

$$g(\overset{3}{f(4)}) = 64a + 8 = 8 \longrightarrow 64a = 0 \longrightarrow a = 0 \quad ✓$$