(سارا السینی‌زاد)

18,75

①

$\log_2^{x-1} \geq 0 \to x-1 \geq 1 \to x \geq 2 \leftarrow D_f$

$-x^2+4x-4 \geq 0 \to -(x^2-4x+4) \geq 0 \to -(x-2)^2 \geq 0 \to x=2$ ✓ فقط $\leftarrow D_g$

$gof(x) \to x\in D_f \,,\, f(x)\in D_g$

↳ $x \geq 2 \,,\, \sqrt{\log_2^{(x-1)}} = 2$

توان ۲ $\log_2^{x-1} = 4 \to x-1 = 2^4 \to x = 17$

2

②

$f(g(x)) = 2(x^2-3x+8)-5 = 2x^2-6x+11$

$g(f(x)) = (2x-5)^2-3(2x-5)+8 = 4x^2-26x+48$

↳ $2x^2-6x+11 = 4x^2-26x+48 \to 2x^2-20x+37 \to S = -\frac{b}{a} = \frac{20}{2} = 10$

$\alpha^2+\beta^2 = S^2-2P = 100-37 = 63 \leftarrow P = \frac{c}{a} = \frac{37}{2}$

2

③

$g(2x) = 5x^2+11 \qquad g(f(x)) = 5x^2+11$

$t = 2x \to x = \frac{t}{2} \to 5(\frac{t}{2})^2+11 = \frac{5}{4}t^2+11 \to g(x) = \frac{5}{4}x^2+$

$g(x-7) = \frac{5}{4}(x-7)^2+11 \to$ کمینه $x=7$

↳ جواب 11

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④

$fog(x) = f(g(x)) = 3g(x)-4$

↳ $3g(x)-4 = 3x^2-6x+3 \to g(x) = x^2-2x+1 \sim g(x) = (x-1)^2$

$gof(x) = g(f(x)) = (f(x)-1)^2 = (3x-4-1)^2 = (3x-5)^2 = \boxed{9x^2-30x+25}$

2

$fog(a) = 0 \qquad f-g\,(a) = 0 \qquad f-g)(a) = 0 \to f(a) = g(a)$

⑤

$(fog)(a) = 0 \qquad f(g(a)) = 0 \qquad f(14) = 0$

$2t = 3t-14 \to t = 14 \qquad f(a) = g(a) = 14 \qquad m = \frac{0-14}{14-a} = \frac{-14}{14-a}$

✓ $y=0 \quad x=14$ ✓ $y=14 \quad x=a \leftarrow y-14 = \frac{14}{14-a}(x-a)$

⑥

$fog(x) \leq gof(x) \quad \xrightarrow{f(g(x))} 9^{\log_3 x} = x^{\log_3 9} = x^2$

$gof \to g(f(x)) = \log_3 9^x = x\log_3 9 = 2x \to$ ↳ $x^2 \leq 2x$

$x^2-2x \leq 0$

$x(x-2) \leq 0$

	0		2	
+		−		+

۳ جواب $\leftarrow 0, 1, 2 \leftarrow [0,2]$

2

(سارا امینی راد)

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$f\left(\frac{x^{۲}-۲}{x}\right) = x^{۲}+x-۴-\frac{۲}{x}+\frac{۴}{x^{۲}} \Rightarrow t=\frac{x^{۲}-۲}{x}=x-\frac{۲}{x}$

$f(t)=x^{۲}+x-۴-\frac{۲}{x}-\frac{۴}{x^{۲}} \Rightarrow t=x-\frac{۲}{x} \rightarrow t^{۲}=x^{۲}-۴+\frac{۴}{x^{۲}}$

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$x^{۲}+\frac{۴}{x^{۲}}=t^{۲}+۴ \rightarrow f(t)=(x^{۲}+\frac{۴}{x^{۲}})+x-۴-\frac{۲}{x}=(t^{۲}+۴)+(x-\frac{۲}{x})-۴$

$f(x)=x^{۲}+x \leftarrow t^{۲}+t$

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$f(x)=\log_{۲}^{x}$

$g(x)=۸x-x^{۲}$

$۸x-x^{۲}>۰ \rightarrow ۰<x<۸$ دامنه $A \rightarrow (۰,۸)$

برد لگاریتم $=R$ $\quad$ $(۰,۸) \xrightarrow{\text{محدود}} [۰,۱۶]$

۲

$B=(-\infty,۴]$ $\quad$ $A\cap B=(۰,۸)\cap(-\infty,۴]=(۰,۴]$

تعداد عضو $\underline{۱,۲,۳,۴} \rightarrow \underline{۴}$

$x(g(x))=f(x)$

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$(x-۱)^{۳}+۲(x-۱)+۱=۱-۳x^{۲}$

$f(\overset{t}{f(x)})=۱-۳f^{۲}(x)$ $\quad$ $\hookrightarrow x^{۳}-۱+۳x-\cancel{۳x^{۲}}+۲x-۲+\cancel{۱}=\cancel{۱}-\cancel{۳x^{۲}}$

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$f(t)=۱-۳t^{۲} \rightarrow f(x)=۱-۳x^{۲}$ $\quad$ $x^{۳}+۵x-۳=۰ \rightarrow$ یک ریشه

۱۰

$f(x)=x-\frac{۴}{x} \rightarrow g\circ f(x)=ax^{۳}+۲x \rightarrow g(۳)=۸$

$g(f(x))=ax^{۳}+۲x$ $\quad$ $f(x)=۳$ $\quad$ $x-\frac{۴}{x}=۳ \rightarrow x^{۲}-۳x-۴=۰$

$(x-۴)(x+۱)=۰$

$g(f(۴))=۶۴a+۸ \rightarrow ۶۴a+۸=۸ \rightarrow a=۰$ $\quad$ ۴ $\quad$ -۱

$g(۳)=g(f(-۱))=-a-۲ \rightarrow -a-۲=۸ \rightarrow -۱۰=a$

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