نام و نام خانوادگی ......آنیسا برسن........ پاسخنامه تشریحی تکلیف شماره ...۵... کلاس ..دوازدهم تجربی A......

۱

$D_{gof} = \{x \mid x \in D_f \ , \ f(x) \in D_g\}$

$\left.\begin{array}{l} x \in D_f \to x \geq 2 \\ f(x) \in D_g \to x = 17 \end{array}\right\} \xrightarrow{\cap} D_{gof} = \{17\}$

$f(x) = \sqrt{\log_2^{x-1}}$ $\qquad g(x) = \sqrt{-x^2+4x-4}$

$x-1>0 \to x>1$ $\qquad -x^2+4x-4 \geq 0$ $\qquad \sqrt{\log_2^{x-1}} = 2 \to \log_2^{x-1} = 4$

$x^2-4x+4 \leq 0$

$\log_2^{x-1} \geq 0 \to x-1 \geq 1 \to x \geq 2$ $\qquad (x-2)^2 \leq 0$ $\qquad x-1 = 16 \quad x = 17$

$D_f = [2, +\infty)$ $\qquad D_g = \{2\}$

۲

$f(x) = 2x-5 \qquad g(x) = x^2-3x+8$

$f(g(x)) = 2(x^2-3x+8)-5 = 2x^2-6x+11$

$g(f(x)) = (2x-5)^2 - 3(2x-5) + 8 = 4x^2-26x+48$

$f(g(x)) = g(f(x)) \longrightarrow 2x^2-6x+11 = 4x^2-26x+48 \to 2x^2-20x+37 = 0 \begin{cases} S = 10 \\ P = \frac{37}{2} \end{cases}$

$\alpha^2+\beta^2 = S^2-2P = 100-37 = 63$

۳

$g(f(x)) = 5x^2+11 \xrightarrow{f(x)=2x} g(2x) = 5x^2+11 \longrightarrow g(x) = 5\left(\frac{x}{2}\right)^2+11$

$g(7-x) = 5\left(\frac{7-x}{2}\right)^2+11$

چون $5\left(\frac{7-x}{2}\right)^2 \geq 0$ بنابراین:

$g(7-x) = 5\left(\frac{7-x}{2}\right)^2+11 \geq 11$

$\min(g(7-x)) = 11$

۴

$f(x) = 3x-4 \qquad f(g(x)) = 3x^2-6x-1 \qquad g(f(x)) = ?$

$f(g(x)) = 3g(x)-4 = 3x^2-6x-1 \longrightarrow g(x) = x^2-2x+1$

$g(f(x)) = (3x-4)^2 - 2(3x-4) + 1 = 9x^2-24x+16-6x+8+1$

$= \boxed{9x^2-30x+25}$

۵

طبق نمودار $= \begin{cases} f(x)-g(x) = -x+5 \\ f(g(x)) = \frac{2}{-a}x+2 \end{cases}$

$2g(x) = 3f(x)-14 \longrightarrow \begin{cases} 3f(x)-2g(x) = 14 \\ f(x)-g(x) = -x+5 \end{cases} \xrightarrow{\times -3}$

$\begin{cases} 3f(x)-2g(x) = 14 \\ -3f(x)+3g(x) = 3x-15 \end{cases}$

$g(x) = 3x-1$

$f(x) = 2x+4$

$f(g(x)) = 2(3x-1)+4 = 6x-2+4 = 6x+2$

$6x+\cancel{2} = \frac{2}{-a}x+\cancel{2} \qquad \frac{2}{-a} = \cancel{6}^{\,3} \rightsquigarrow a = \frac{-1}{3}$

$f(x)=9^x \quad g(x)=\log_3^x \rightsquigarrow D_g = x>0 \quad D_f = \mathbb{R}$

$f(g(x)) = 9^{\log_3^x} = x^{\log_3^9} = x^2 \xrightarrow{\text{شرط دامنه}} x>0$

$g(f(x)) = \log_3^{9^x} = \log_3^{3^{2x}} = 2x \xrightarrow{\text{شرط دامنه}} \mathbb{R}$ ۶

$\xrightarrow{\cap} x>0$ (I)

$fog(x) \leqslant gof(x) \to x^2 \leqslant 2x \to x^2-2x \leqslant 0 \to x(x-2)\leqslant 0$

$\begin{array}{ccccc} & 0 & & 2 & \\ + & & - & & + \end{array}$ $[0,2]$ (II)

$I, II \rightsquigarrow (0,2] \to$ شامل دو عدد صحیح است

$f\left(\frac{x^2-2}{x}\right) = x^2+x-4-\frac{2}{x}+\frac{4}{x^2} \to f\left(x-\frac{2}{x}\right) = \underbrace{x^2-4+\frac{4}{x^2}}_{\left(x-\frac{2}{x}\right)^2} + \left(x-\frac{2}{x}\right)$

$f\left(x-\frac{2}{x}\right) = \left(x-\frac{2}{x}\right)^2 + \left(x-\frac{2}{x}\right)$ ۷

$\Rightarrow f(x)=x^2+x$

$f(x)=\log_2^x \quad g(x)=-x^2+8x \rightsquigarrow D_f=(0,+\infty) \quad D_g=\mathbb{R}$

$A = D_{fog} = \{x \mid x\in D_g,\ g(x)\in D_f\}$ $\quad x\in\mathbb{R}$ I

$g(x)\in(0,+\infty) \to -x^2+8x>0$ $\quad \begin{array}{ccccc} & 0 & & 8 & \\ - & & + & & - \end{array}$ $x\in(0,8)$ II

$D_{fog} = I \cap II = (0,8) = A$ ۸

$fog = \log_2^{-x^2+8x} \quad -x^2+8x \to ext\Big|^{4}_{16} \rightsquigarrow R_{fog} = (-\infty,4] = B$

$A\cap B = (0,4] \to$ شامل چهار عدد صحیح

$f(f(x)) = 1-3f^2(x) \to f(x) = 1-3x^2 \quad g(x)=x-1 \quad h(x)=x^3+2x+1$

$h(g(x)) = f(x) \to (x-1)^3+2(x-1)+1 = 1-3x^2$

$x^3-\cancel{3x^2}+3x-\cancel{1}+2x-2+\cancel{1} = 1-\cancel{3x^2} \to x^3+5x-3=0$ ۹

دارای یک ریشه

$f(x)=x-\frac{4}{x} \quad g(f(x))=ax^3+2x \quad g(3)=8$

$f(x)=3 \to x-\frac{4}{x}=3 \xrightarrow{\times x} x^2-3x-4=0 \to x=-1$ یا 4

$g(f(4))=8 \to 64a+8=8 \to 64a=0 \to a=0$ ۱۰

$g(f(-1))=8 \to -a-2=8 \to a=-10$