الیسا خان محمدی

① $D_{gof} = \{ x \in D_f \ , \ f(x) \in D_g \}$

(1)

$f(x) = \sqrt{\log_2^{x-1}}$ $\quad D_f = \quad x-1>0 \quad x>1 \quad \log_2^{x-1} \geq 0$

$x-1 \geq 1 \quad \boxed{x \geq 2}$

(2) $g(x) = \sqrt{-(x-2)^2} \longrightarrow D_g . \sqrt{\log_2^{x-1}} = 2$

$x = 2 \quad \log_2^{x-1} = 4 \quad x-1 = 16 \quad \boxed{x = 17}$

$D_{gof} = \{17\}$

② $fog(x) = gof(x)$

$2(x^2 - 3x + 8) - 5 = (2x-5)^2 - 3(2x-5) + 8 \rightarrow 4x^2 + 25 - 20x - 6x + 15 + 8$

$2x^2 - 6x + 11 = 4x^2 - 26x + 48 \Longrightarrow 2x^2 - 20x + 37 \qquad S = \frac{20}{2} = 10$

$P = 37/2$

$\alpha^2 + \beta^2 = S^2 - 2P = 100 - 37 = 63$

③ $g(f(x)) = 5x^2 + 11 \qquad f(x) = 2x \qquad g(2x) = 5x^2 + 11$

$2x = t \qquad x = \frac{t}{2}$

$g(t) = 5\left(\frac{t}{2}\right)^2 + 11$

$g(t) = \frac{5t^2}{4} + 11 \longrightarrow g(x-7) = \frac{5(x-7)^2}{4} + 11 \quad \min : x = 7$

$g(x-7) = 11$

④ $f(g(x)) = 3x^2 - 6x - 1 \quad\to\quad g(x) = \frac{3x^2 - 6x + 3}{3}$

$3g(x) - 4 = 3x^2 - 6x - 1 \qquad g(x) = x^2 - 2x + 1 \quad (x-1)^2$

$g(f(x)) = (3x - 4 - 1)^2 = (3x-5)^2 = 9x^2 - 30x + 25$

⑤

$$\begin{cases} f(x) - g(x) = -x + 5\,x+2 \\ 2g(x) = 3f(x) - 14 \end{cases}$$

$2f(x) - 2g(x) = -2x + 10$

$2g(x) - 3f(x) = -14$

$f(x) = 2x + 4 \longrightarrow g(x) = 3x - 1$

$f\circ g(x) = f(g(x)) = 2(3x-1) + 4 = 6x + 2$

$f\circ g(x) = -\frac{2}{a}x + 2 = 6x + 2 \quad \boxed{a = -\frac{1}{3}}$

⑥

$f(x) = 9^x \qquad f\circ g(x) < g\circ f(x)$

$g(x) = \log_3 x \qquad 9^{\log_3 x} < \log_3 9^x$

$x^2 \leftarrow x$ $\qquad x\log_3 9 = 2x$

$x^2 < 2x \qquad x^2 - 2x < 0 \qquad \boxed{0 < x < 2}$

$x \in (0,2] \leftarrow$ پس $D_g = x > 0$ اما $D_f = \mathbb{R}$ اما شرط دامنه داریم

۲ عدد صحیح ۱ و ۲

⑦

$f\left(\frac{x^2-2}{x}\right) = \underbrace{x^2 - 4 + \frac{4}{x^2}}_{x^2} + \underbrace{x - \frac{2}{x}}_{x} \qquad f(x) = x^2 + x$

$x - \frac{2}{x}$

$x - \frac{2}{x} = t \rightarrow t^2 = x^2 - 4 + \frac{4}{x^2}$

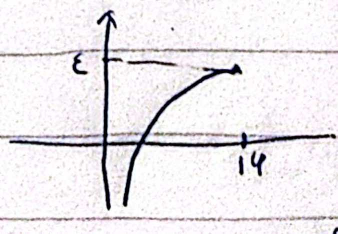
⑧

$f(x) = \log_2 x \qquad A = \{x \in D_g \,,\, g(x) \in D_f\}$

$g(x) = -x^2 + 8x \qquad D_g = \mathbb{R} \,,\, D_f = x > 0 \rightarrow -x^2 + 8x > 0$

$B = \log_2 (-x^2+8x)$

$A \cap B = (0, 4]$



$0 < x < 8$

$B = (-\infty, 4]$

$1, 2, 3, 4$

۴ عدد

← با طول جا به جا تغییر کردیم

برای گروه ۶ هم ۷

s.am

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$f(\underbrace{f(x)}_{t}) = 1 - 3\underbrace{f^2(x)}_{t^2}$ $\quad$ $f(x) = 1 - 3x^2$

$(x-1)^3 + 2(x-1) + 1 = 1 - 3x^2 \quad \rightarrow \quad x^3 + 5x - 3 = 0$

$x^3 + 3x - \cancel{3x^2} + 2x + \cancel{1} - \cancel{3} = \cancel{1} - \cancel{3x^2}$

$b^2 - 3ab = 0 - 3(1)(5)$

ریشه دارد ۱ ← یکنوا $\xleftarrow{\text{پس}}$ $-15 < 0$

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$f(x) = x - \frac{4}{x} \qquad g(f(x)) = ax^3 + 2x \qquad g(3) = 8$

$ax^3 + 2x = g(3) = 8 \qquad 3 = x - \frac{4}{x}$

$x^2 - 3x - 4 = 0 \qquad (x - 4)(x + 1) = 0 \rightarrow x = -1, \; x = 4$

$a(-1)^3 - 2 = 8$

$-a = 10 \rightarrow a = -10$

$a(4)^3 + 8 = 8 \rightarrow a = 0$