اروین فنودیل

17,75

1. $gof(x) \longrightarrow x\in D_f \ , \ f(x)\in D_g$

$D_g: \quad -x^2+4x-4\ge 0 \longrightarrow -(x-2)^2\ge 0 \longrightarrow x=2 \longrightarrow f(x)=2$

$\longrightarrow \log_2^{x-1}=4 \qquad x-1=16 \qquad \boxed{x=17} \longrightarrow D_{fog}=\{17\}$

2

2. $fog=2(x^2-3x+8)-5 = 2x^2-6x+16-5 = 2x^2-6x+11$

$gof=(2x-5)^2-3(2x-5)+8 = 4x^2-20x+25-6x+15+8 = 4x^2-26x+48$

$2x^2-6x+11=4x^2-26x+48 \longrightarrow 2x^2-20x+37=0$

$\alpha^2+\beta^2=S^2-2P= 100-2(\frac{37}{2}) = \boxed{63}$

2

3. $gof(x)=5x^2+11 \longrightarrow g(f(x))=5x^2+11 \quad \frac{5}{4}(2x)^2+11 = g(x)$

$g(x-7)=\frac{5}{4}(x-7)^2+11 \xrightarrow{\text{کمترین مقدار}} 11 \longrightarrow \boxed{11}$

2

4. $f(x)=3x-4 \qquad fog=3x^2-6x-1 \Leftrightarrow f(g(x))=$

$gof(x)=? \qquad f(g(x))=3(g(x))-4=3x^2-6x-1$

$g(x)=\frac{3x^2-6x+3}{3}=x^2-2x+1 \longrightarrow$

2

$\longrightarrow g(x)=(x-1)^2 \qquad gof(x)=(3x-4-1)^2=(3x-5)^2=9x^2-30x+25$

5. $f-g=-x+5 \qquad f(x)-g(x)=-x+5 \longrightarrow 3f(x)-3g(x)=-3x+15$

$2g(x)-3f(x)=-14 \longrightarrow -3f(x)+2g(x)=-14$

$-g(x)=-3x+1 \longrightarrow g(x)=3x-1$

$6x-2-3f(x)=-14$

$3f(x)=6x+12$

$f(x)=2x+4 \longrightarrow f(g(x))=2(3x-1)+4=6x-2+4=6x+2$

$fog=6x+2=0 \qquad x=-\frac{2}{6} \qquad \boxed{x=-\frac{1}{3}}=a$

2

6. $f(x)=9^x \quad g(x)=\log_3^x \qquad fog(x)\le gof(x)$

$9^{\log_3^x}\le \log_3^{9^x} \longrightarrow x^{\log_3^9}\ge x\log_3^9$

$x(x-2)\le 0$ $0 \quad 2$: $+ \ | \ - \ | \ +$

$\longrightarrow x^2\le 2x \longrightarrow x^2-2x\le 0$

$\longrightarrow \cancel{D_f}\ x=[0 \quad 2]$

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1,75

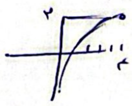
(۷) $f(x-\frac{2}{x}) = (x-\frac{2}{x})^2 + (x-\frac{2}{x}) \rightarrow f(x) = x^2 + x$ ✓ (۲)

(۸) $f(x) = \log_2 x \quad g(x) = 8x - x^2 \quad fog(x) = D_f, R_f \quad A \cap B$

(۱)

$fog = \log_2^{8x-x^2} \longrightarrow 8x - x^2 \geq 0 \quad x(8-x) \geq 0$ $\quad D_f = (0 \quad 8)$ ✓

$\max\ 8x - x^2 : \frac{-8}{2} = 4 \quad \boxed{\log_2 4 = 2}$



$\longrightarrow R_f = (-\infty \quad 2)$

$R = (-\infty, 4]$

$R_f \cap D_f = (0 \quad 2) \quad A \cap B = (0, 4] \longrightarrow$ ۴ عضو صحیح

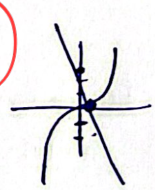
(۹) $fof = 1 - 3f^2(x) \quad g(x) = x - 1 \quad h(x) = x^3 + 3x + 1$

$f(f(x)) = 1 - 3f^2(x)$

$\rightarrow f(x) = 1 - 3x^2$

$(x-1)^3 + 3(x-1) + \cancel{1} = 1 - 3x^2$

$x^3 - \cancel{3x^2} + 3x - 1 + 3x - 3 = \cancel{-3x^2} \quad x^3 + 6x - 4 = 0 \quad x^3 = 4 - 6x$ ✓ (۲)



۱ تقاطع دارد ✓

(۱۰) $f(x) = x - \frac{4}{x} \quad gof = ax^3 + 2x \quad g(3) = 8$

$g(f(x)) = ax^3 + 2x$

$f(x) = 3 \quad x - \frac{4}{x} = 3 \quad x^2 - 3x - 4 = 0 \quad (x-4)(x+1) = 0 \rightarrow x = -1,\ x = 4$ ✓ (۱)

$g(f(4)) = 64a + 8 \quad g(\underbrace{f(-1)}_{3}) = -a - 2 \quad -a - 2 = 64a + 8 \quad 65a = -10 \quad a = \frac{-10}{65} = \boxed{\frac{-2}{13}}$

$x = 4 \longrightarrow 64a + 8 = 8 \longrightarrow a = 0$

$x = -1 \longrightarrow -a - 2 = 8 \longrightarrow a = -10$

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