

$$D_{g \circ f} = \{x \in D_f : f(x) \in D_g\}$$

$$f(x) = \sqrt{\log_p(x-1)}$$

$$g(x) = \sqrt{-x^2 + 4x - 4}$$

$$x \in D_f \rightarrow x-1 > 0 \quad \boxed{x > 1} \quad \text{I}$$

$$\log_p(x-1) \geq 0 \quad x-1 \geq 1 \quad \boxed{x \geq 2} \quad \text{II}$$

$$f(x) \in D_g$$

$$-x^2 + 4x - 4 \geq 0$$

$$x^2 - 4x + 4 \leq 0$$

$$(x-2)^2 \leq 0 \quad x=2$$

$$\sqrt{\log_p(x-1)} = p$$

$$\log_p(x-1) = p^2$$

$$x-1 = 16$$

$$\boxed{x=17} \quad \text{III}$$

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$I \cap II \cap III$
 $\boxed{x=17}$

$$f \circ g(x) = \sqrt{(x^2 - 13x + 11) - 4} = \sqrt{x^2 - 13x + 7}$$

$$g \circ f(x) = (px - 4)^2 - 3(px - 4) + 1 = p^2x^2 + 4p^2 - 3p^2x - 4p + 1 = p^2x^2 - 3p^2x + 4p^2 - 4p + 1$$

$$f \circ g(x) = g \circ f(x)$$

$$p^2x^2 - 13px + 11 = p^2x^2 - 3p^2x + 4p^2 - 4p + 1$$

$$a^2 + b^2 = p^2 \rightarrow s^2 - 2p$$

اگر a و b
ها باشند

$$s^2 - 2p = 100 - 3p = \boxed{97}$$

$$p = \frac{c}{a}$$

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$$s = \frac{y_0}{p} = 10 \quad p = \frac{3y_0}{p}$$

$$f(x) = px$$

$$px = t$$

$$x = \frac{t}{p}$$

$$g \circ f(x) = \omega x^p + 11$$

$$\omega \left(\frac{t}{p}\right)^p + 11 = \omega \left(\frac{t^p}{p^p}\right) + 11 = \frac{\omega t^p}{p^p} + 11$$

$$g(x) = \frac{\omega}{p} x^p + 11$$

$$g(x-v) = \frac{\omega}{p} (x-v)^p + 11 = \frac{\omega}{p} (x^p + 49 - 14x) + 11 = \frac{\omega}{p} x^p + \frac{49\omega}{p} - \frac{14\omega}{p} x + 11$$

$$= \frac{\omega}{p} x^p - \frac{3\omega}{p} x + \frac{49\omega + 11p}{p}$$

$$x=v \rightarrow \left(\frac{\omega}{p} \times 49\right) - \left(\frac{3\omega \times v}{p}\right) + \frac{49\omega + 11p}{p} = \frac{49\omega}{p} - \frac{3\omega v}{p} + \frac{49\omega + 11p}{p} = \frac{49\omega - 3\omega v + 49\omega + 11p}{p} = \frac{98\omega - 3\omega v + 11p}{p}$$

$$= \frac{98\omega}{p} = \boxed{11}$$

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$$f(x) = 3x - 4 \quad f \circ g(x) = 3x' - 4x - 1 \quad g \circ f(x) = ?$$

$$3g - f = 3x' - 4x - 1 \quad 3g = 3x' - 4x + 1 \quad g(x) = x' - 4x + 1$$

$$(3x - 4)' - 4(3x - 4) + 1 = 9x' + 12 - 12x - 4x + 1 + 1 = 9x' - 10x + 14$$

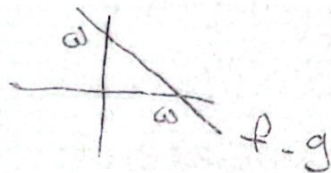
$$g \circ f(x) = 9x' + 10x + 14$$

$$f(x) = ax + b \quad g(x) = cx + d \quad \text{نسبت } f \text{ و } g \text{ با هم برابر است (تابع خطی است)} \quad \textcircled{a}$$

$$3g(x) = 3f(x) - 14 \quad 3cx + 3d = 3ax + 3b - 14 \quad 3c = 3a \quad 3d = 3b - 14$$

$$f - g = (a - c)x + (b - d)$$

$$f - g = -x + a$$



$$\text{نسبت} = -1 \quad -x + a$$

$$a - c = -1 \quad b - d = a$$

$$a = c - 1 \quad 3a = 3c \quad 3(c - 1) = 3c$$

$$3c - 3 = 3c$$

$$c = 3$$

$$a = 2$$

$$b = d + a$$

$$3d = 3(d + a) - 14$$

$$3d = 3d + 12 - 14$$

$$3d = 3d + 1$$

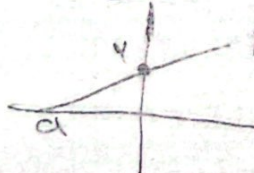
$$d = -1$$

$$b = 2$$

$$f(x) = 3x + 2$$

$$g(x) = 3x - 1$$

$$f \circ g(x) = 3(3x - 1) + 2 = 9x - 3 + 2 = 9x - 1$$



$$9x - 1 = 0 \quad 9x = 1$$

$$x = \frac{1}{9} = \frac{1}{3}$$

$$a = \frac{1}{3}$$

جواب آخر

$$f(x) = 9^x \quad g(x) = \log_9 x$$

$$f \circ g(x) = 9^{\log_9 x}$$

$$9^{\log_9 x} = x^{\log_9 9}$$

$$f \circ g(x) = x^1$$

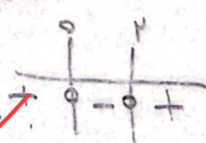
$$g \circ f(x) = \log_9 9^x$$

$$= x \log_9 9 = x$$

$$f \circ g(x) \leq g \circ f(x) \rightarrow x^1 \leq x$$

$$x^1 - x \leq 0$$

$$x(x - 1) \leq 0$$



جواب آخر

$$\{0, 1\}$$

$$x^3 =$$

$$f\left(\frac{x^2-1}{x}\right) = x^2 + x - \frac{1}{x} + \frac{1}{x^2} \quad f(x) = ?$$

$$f\left(x - \frac{1}{x}\right) = \left(x - \frac{1}{x}\right) + \left(x^2 + \frac{1}{x^2} - f\right) \quad x - \frac{1}{x} = t$$

$$f(t) = t + (t^2) \quad \boxed{f(x) = x^2 + x} \quad \checkmark \quad (5)$$

$$A = D_{f \circ g} \rightarrow x \in D_g : g(x) \in D_f \quad D_g = \mathbb{R} \quad D_f \rightarrow x > 0 \quad (\Delta)$$

$$g(x) > 0 \rightarrow 1x - x^2 > 0 \quad x^2 - 1x < 0 \quad x(x-1) < 0$$

$$A = \{1, 2, 3, 4, 5, 6, 7\}$$

$$f \circ g(x) = \log_p(1x - x^2) \quad R_{f \circ g} = B$$

$$\frac{-x^2 + 1x}{\text{Max}} \rightarrow \frac{-b}{2a} = \frac{-1}{-2} = \frac{1}{2}$$

$$(-\infty, \frac{1}{2}]$$

$$\log_p(1x - x^2)$$

چون در $\log$ قرائی غیر صفر باشد؛
تغییرات $1x - x^2$ می شود $(0, \frac{1}{2}]$

$$R_{f \circ g(x)} \rightarrow (-\infty, \frac{1}{2}] \quad B = (-\infty, \frac{1}{2}] \quad A \cap B \xrightarrow{x \in \mathbb{Z}}$$

$$\{1, 2, 3, 4\} \Rightarrow \text{با ۴ عضو صحیح است.} \quad \checkmark$$

$$f \circ f(x) = f(f(x)) \quad x \xrightarrow{f} f(x) \xrightarrow{f} 1 - 3f^2(x)$$

$$f(x) = 1 - 3x^2 \quad h(g(x)) = (x-1)^2 + 2(x-1) + 1 = x^2 - 3x^2 + 3x - 1$$

$$+ 2x - 1 + 1 = x^2 - 3x^2 + 3x - 1 \quad h(g(x)) = f(x) \quad (5)$$

$$x^2 - 3x^2 + 3x - 1 = 1 - 3x^2$$

$$x^2 + 3x - 3 = 0 \quad \checkmark \quad x \approx \frac{56}{100} \quad \checkmark \quad \text{یک دوازدهم}$$

$$f(x) = x - \frac{f}{x} \quad g \circ f(x) = ax^3 + px \quad g(3) = 1 \quad a = p$$

$$x - \frac{f}{x} = 3 \quad \frac{x^2 - f}{x} = 3 \quad x^2 - f = 3x \quad x^2 - 3x - f = 0 \quad (x-3)(x+1) = 0$$

$$f(x) \xrightarrow{x=3} 3 \rightarrow g \circ f(x) = 1$$

$$9a + 1 = 1$$

$$a = 0 \quad \checkmark$$

$$f(x) \xrightarrow{x=-1} 3 \rightarrow g \circ f(x) = 1$$

$$-a - 1 = 1$$

$$-a = 2 \quad a = -2 \quad \checkmark$$

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