(A) دوازدهم

۲۰

سیدپارسا شهرکی‌نیا

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$gof = \{x \in D_f ,\ f(x) \in D_g\}$ $\qquad f(x) = \sqrt{\log_2^{(x-1)}}$

$g(x) = \sqrt{-x^2+4x-4}$

$D_f \to x-1>0 \to x>1 \ (1) ,\ \log_2^{x-1} \geq 0 \to x-1 \geq 1 \to x \geq 2 \ (2)$

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$D_g \to -x^2+4x-4 \geq 0 \to x^2-4x+4 \leq 0 \to (x-2)^2 \leq 0 \to x=2$

$f(x) \in D_g \to \sqrt{\log_2^{x-1}} = 2 \to \log_2^{x-1} = 4 \to x-1=16 \to x=17 \ (3)$

$(1) \cap (2) \cap (3) = x = 17$

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$f(x) = 2x-5 ,\ g(x) = x^2-3x+8$

$fog(x) = 2(x^2-3x+8)-5 \to \underline{2x^2-6x+11}$

$gof(x) = (2x-5)^2-3(2x-5)+8 \to 4x^2-20x+25-6x+15+8 = \underline{4x^2-26x+48}$

$fog(x) = gof(x) \to 2x^2-6x+11 = 4x^2-26x+48 \to 2x^2-20x+37=0$

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$\alpha^2+\beta^2 = ? \longrightarrow S^2-2P \to S = -\frac{b}{a} = \frac{20}{2} = 10 ,\ P = \frac{c}{a} = \frac{37}{2}$

$S^2 = 100 ,\ 2P = 37 \to 100-37 = 63$

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$gof(x) = 5x^2+11 \qquad f(x) = 2x \to 2x = t \to x = \frac{t}{2}$

$gof(t) = 5\left(\frac{t}{2}\right)^2+11 \to \frac{5t^2}{4}+11 \to gof(x) = \frac{5}{4}x^2+11$

$g(x-7) \to \frac{5}{4}(x-7)^2+11 = \frac{5}{4}(x^2-14x+49)+11 \to \frac{5}{4}x^2 - \frac{35}{2}x + \frac{289}{4} \to$

$\to -\frac{b}{2a} = \frac{\frac{35}{2}}{\frac{5}{2}} = 7 \xrightarrow{x=7} \left(\frac{5}{4}\times 49\right) - \left(\frac{35}{2}\times 7\right) + \frac{289}{4} = \frac{245-490+289}{4}$

۲ $\qquad \frac{44}{4} = 11$

دوازدهم (A)

پریا شیرازی نیا

۴) $f(x)=3x-4$, $f\circ g(x)=3x^2-6x-1$ $\quad g\circ f(x)=?$

$3g-4=3x^2-6x-1 \to 3g=3x^2-6x+3 \to g(x)=x^2-2x+1$

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$g\circ f(x)=(3x-4)^2-2(3x-4)+1=9x^2-24x+16-6x+9 \to$

$\to \boxed{9x^2-30x+25}$

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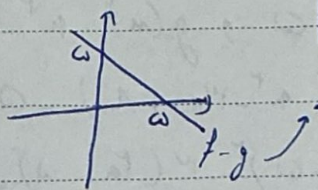
۵) نمودار $f-g$ خطی است پس هر دو تابع خطی اند

$f(x)=ax+b$

$g(x)=cx+d$

$2g(x)=3f(x)-14 \qquad 2cx+2d=3ax+3b-14$ $\quad \begin{cases} 2c=3a \\ 2d=3b-14 \end{cases}$



$f-g=(a-c)x+(b-d)$

$f-g=-x+5$

$a-c=-1 \to a=c-1 \xrightarrow{3a=2c} 3(c-1)=2c \to \boxed{c=3} , \boxed{a=2}$

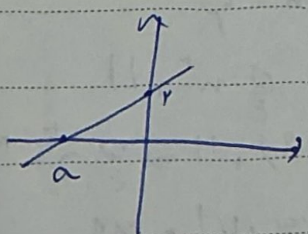
$b-d=5 \to b=5+d \xrightarrow{2d=3b-14} 2d=3(5+d)-14 \to \boxed{d=-1}$

$\boxed{b=4}$

$f(x)=2x+4 \to f\circ g(x)=2(3x-1)+4=6x+2$

$g(x)=3x-1$

$6x+2=0 \to x=\boxed{-\frac{1}{3}=a}$



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۶ـ $f(x)=9^x \ , \ g(x)=\log_3 x \ , \ fog(x)\le gof(x)$

$fog(x)=9^{\log_3 x} \to x^{\log_3 9} \to x^2$

$gof(x)=\log_3 9^x \to x\log_3 9 \to 2x$

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$fog(x)\le gof(x) \to x^2\le 2x \to x^2-2x\le 0 \to x(x-2)\le 0$

$\begin{array}{ccccc} & 0 & & 2 & \\ + & | & - & | & + \end{array}$

$[0,2] \to$ تعداد اعضای صحیح $\to 3$

$\{0,1,2\}$

۷ـ $f\left(\frac{x^2-2}{x}\right)=x^2+x-4-\frac{2}{x}+\frac{4}{x^2} \qquad f(x)=?$

$f\left(x-\frac{2}{x}\right)=\left(x-\frac{2}{x}\right)+\left(x^2+\frac{4}{x^2}-4\right)$

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$x-\frac{2}{x}=t \to f(t)=t+t^2 \to \boxed{f(x)=x^2+x}$

۸ـ $f(x)=\log_2 x \ , \ g(x)=8x-x^2 \qquad A=D_{fog} \ , \ B=R_{fog}$

$A=D_{fog} \to \{x\in D_g \ , \ g(x)\in D_f\} \to D_g=\mathbb{R} \ , \ D_f \to x>0 \to$

$\to g(x)>0 \to 8x-x^2>0 \to x^2-8x<0 \to x(x-8)<0$

$\boxed{A=(0,8)} \qquad \begin{array}{ccccc} & 0 & & 8 & \\ + & | & - & | & + \end{array}$

$B=R_{fog} \to fog(x)=\log_2^{8x-x^2} \longrightarrow 8x-x^2 \ \max \ , \ \begin{cases} \frac{-b}{2a}=4 \\ y_{max}=16 \end{cases} \to$

$\to (-\infty,16] \to \log_2^{8x-x^2} \to$ باید بزرگتر از صفر باشد پس جواب برابر با $(0,16]$ و داریم $\longrightarrow$ از $(0,16]$ لوگ آن بگیریم

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$\boxed{B=(-\infty,4]} \leftarrow (-\infty,4]$

$\boxed{A\cap B \to \{1,2,3,4\}}$

جواب ← ۴ عضو صحیح دارد

$fof(x) = 1 - 3f^2(x) \qquad g(x) = x - 1 \qquad h(x) = x^3 + 2x + 1$ — 9

$fof(x) = f(f(x)) \rightarrow x \xrightarrow{f} f(x) \xrightarrow{f} 1 - 3f^2(x) \longrightarrow$

$\rightarrow f(x) = 1 - 3x^2$

$h(g(x)) = (x-1)^3 + 2(x-1) + 1 \rightarrow x^3 - 3x^2 + 3x - 1 + 2x - 2 + 1 \rightarrow$

$g(x) = x - 1$

$\rightarrow x^3 - 3x^2 + 5x - 2 \rightarrow \quad h(g(x)) = f(x)$

$$x^3 - 3x^2 + 5x - 2 = 1 - 3x^2$$

$$x^3 + 5x - 3 = 0$$

$$x \simeq \frac{54}{100}$$

2

$f(x) = x - \frac{4}{x} \qquad gof(x) = ax^3 + 2x \qquad g(3) = 8 \qquad a = ?$ — 10

$x - \frac{4}{x} = 3 \rightarrow x^2 - 3x - 4 = 0 \rightarrow (x-4)(x+1) = 0 \rightarrow x = 4,\ x = -1$

2

$f(x) \xrightarrow{x=4} 3 \rightarrow gof(x) = 8 \rightarrow 64a + 8 = 8 \rightarrow a = 0$

$\xrightarrow{x=-1} 3 \rightarrow gof(x) = 8 \rightarrow -a - 2 = 8 \rightarrow -a = 10 \rightarrow a = 10$