$f(x)=\sqrt{\log_2^{x-1}} \Rightarrow \log_2^{x-1} \geq 0$ — ۱

$x-1 \geq 1 \Rightarrow x \geq 2$

$x-1>0 \Rightarrow x>1$

$\Rightarrow x \geq 2$

$g(x)=\sqrt{-x^2+4x-4}$

$\Rightarrow -x^2+4x-4 \geq 0$

$\underset{-\ \ 0\ \ -}{2} \Rightarrow x=2$

$D_{gof}=\{x \in D_f , f(x) \in D_g\}$

$x \geq 2 \ \mathrm{I}$

$\sqrt{\log_2^{x-1}}=2 \Rightarrow \log_2^{x-1}=4 \Rightarrow x-1=16 \Rightarrow x=17 \ \mathrm{II}$

$\mathrm{I} \cap \mathrm{II} = x=\boxed{17}$

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۲ — $2(x^2-3x+8)-5=(2x-5)^2-3(2x-5)+8 \Rightarrow 2x^2-6x+11=4x^2+25-20x-6x+15+8$

$\Rightarrow 2x^2-10x+37=0$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=5^2-2\times\frac{37}{2}=25-37=\boxed{-12}$

$2x=t \Rightarrow x=\frac{t}{2}$ $\quad g(t)=5\left(\frac{t}{2}\right)^2+11=\frac{5t^2}{4}+11$ — ۳

$g(x-7)=\frac{5}{4}(\underset{\downarrow\, 7}{x-7})^2+11=\boxed{11}$

$3g-4=3x^2-6x-1 \Rightarrow 3g=3x^2-6x+3 \Rightarrow g(x)=x^2-2x+1$ — ۴

$gof(x)=(3x-4)^2-2(3x-4)+1=9x^2+16-24x-6x+8+1=9x^2-30x+25$

$\boxed{=(3x-5)^2}$

$g(x)=\frac{3}{2}f(x)-7$ $\quad f(x)-g(x)=-x+5 \Rightarrow f(x)-\left(\frac{3}{2}f(x)-7\right)=-x+5$ — ۵

$\Rightarrow -\frac{1}{2}f(x)+7=-x+5 \Rightarrow f(x)=2x+4$ $\quad g(x)=\frac{3}{2}(2x+4)-7=3x-1$

$fog(x)=2(3x-1)+4=6x+2$ $\quad 6x+2=0 \Rightarrow x=-\frac{1}{3}$ $\quad \boxed{a=-\frac{1}{3}}$

۶-

$$fog_{(x)} = 9^{\log_3^x} = x^{\log_3^9} = x^2$$

$$gof_{(x)} = \log_3^{9^x} = x \times \log_3^9 = 2x$$

$$\Rightarrow x^2 < 2x \Rightarrow x^2 - 2x < 0$$

$$x(x-2) < 0$$

$$\begin{array}{ccccc} & 0 & & 2 & \\ + & | & - & | & + \end{array} \Rightarrow x \in (0,2)$$

$\boxed{\text{۲ عضو صحیح}}$

۷-

$$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2} \Rightarrow f\left(x - \frac{2}{x}\right) = x - \frac{2}{x} + \left(x - \frac{2}{x}\right)^2$$

$$\Rightarrow f_{(x)} = \boxed{x^2 + x}$$

۸-

$$f_{(x)} = \log_2^x \Rightarrow x > 0 \qquad g_{(x)} = 8x - x^2 \Rightarrow x \in R$$

$$D_{fog} = \{x \in D_g \;,\; g_{(x)} \in D_f\} \rightarrow -x^2 + 8x > 0$$

$$x \in R \;\; \text{I} \qquad 0 < x < 8 \;\; \text{II} \Rightarrow \text{I} \cap \text{II} = x \in (0,8) \quad A$$

$$g_{(x)} = -x^2 + 8x \Rightarrow g_{(x)} \in (-\infty, 16] \qquad fog_{(x)} = \log_2^{-x^2+8x \rightarrow (0,16]}$$

$$\Rightarrow R_{fog_{(x)}} = (-\infty, 4] \quad B \qquad A \cap B = [0,4]$$

$\boxed{\text{۵ عضو صحیح}}$

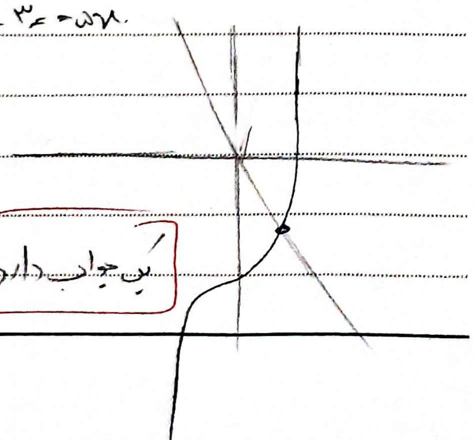
۹-

$$f(f_{(x)}) = 1 - 3f^2_{(x)} \Rightarrow f_{(x)} = 1 - 3x^2$$

$$h(g_{(x)}) = (x-1)^3 + 2(x-1) + 1 = x^3 - 1 - 3x(x-1) + 2x - 2 + 1 = x^3 - 1 - 3x^2 + 3x + 2x - 2 + 1$$

$$= x^3 - 3x^2 + 5x - 2$$

$$x^3 - \cancel{3x^2} + 5x - 2 = 1 - \cancel{3x^2} \Rightarrow x^3 + 5x - 3 = 0 \Rightarrow x^3 - 3 = -5x$$



$\boxed{\text{یک جواب دارد}}$

۱۵-

$x - \frac{4}{x} = 3 \Rightarrow x^2 - 3x - 4 = 0 \rightarrow x = -1 , 4$

$g(f(x+1)) = ax^3 + 2x \xrightarrow{x=4} 64a + 8 = 8 \rightarrow a = 0$

$\xrightarrow{x=-1} -a - 2 = 8 \rightarrow a = -10$